Assessing the evolution of learning capabilities and disorders with graphical exploratory analysis of surveys containing missing and conflicting answers

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Abstract:

Tests and surveys have become common for acquiring information about educational results. On the one hand, certain learning disorders can be diagnosed by solving tests at different development stages of children. Comparing the results of two tests separated in time, it is possible to determine whether the skills of a child are evolving at the proper rate. On the other hand, a different kind of test, the knowledge survey, serves the teacher for determining the student's prerequisite skills and for measuring the fulfillment of the learning outcomes of the course. In both cases the methodologies for analyzing the data are similar as it is needed to compare two distinct sets of answers to the same set of questions, searching for similarities between individuals and studying also the evolution of the answers.

The statistical treatment of this problem is well known, except for the case where there are missing or conflicting answers in the tests. In this paper we propose to extend certain graphical exploratory analysis techniques to this last situation. Our objective is to project the data in a map, where each individual will be placed depending on his/her knowledge profile, allowing the teacher to identify groups with similar background problems, segment heterogeneous groups and perceive the evolution of the learning skills or the abilities acquired during the course. The main innovation of our approach consists in regarding the answers to the tests as imprecise data. We will consider that either a missing or unknown answer, or a set of conflictive answers to a survey, is aptly represented by an interval or a fuzzy set. This representation causes that each individual in the map is no longer a point but a figure, whose shape and size determine the coherence of the answers and whose position with respect to its neighbors determine the similarities and differences between the individuals.

1. Introduction

Sensible measures for the evolution of students' capabilities are important in order to choose adequate teaching methods. At the beginning of a course, the prerequisite skills of a sometimes heterogeneous student body must match the instructional

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approach. After the course ends, the evaluation of the learning outcomes should also consider the differences between the initial preparation of the students, so the impact of the teaching methodology for each kind of student can be precisely assessed.

From another point of view, evaluating the learning outcomes with respect to the prerequisite skills is arguably akin to the problem of measuring the evolution of certain learning disorders. For example, let us consider the diagnosis of dyslexia in children. This problem is detected with non-writing based tests, measuring capabilities such as verbal comprehension, logic reasoning and sensory-motor skills [15]. As we will explain later in this paper, it is not easy to detect dyslexia in early childhood, as the natural differences between the skills of the children mask the symptoms. However, if the tests are done yearly, then each child passes through different development stages and interesting information can be obtained if the changes between two consecutive tests are analyzed. Then again, a sensible measure of the evolution of the learning capabilities is needed.

In both cases, the information is acquired by means of questionnaires. Generally speaking, a questionnaire is intended to measure a number of latent or hidden variables, whose value is indirectly determined by averaging the answers to many different questions related to the observable variables, or items. In the following, we will use the term *multi-item value* to refer to the set of items containing all the information conveyed by the questionnaire about the value of a latent variable. We will also assume that the latent variables measure the capability of an individual for solving certain kind of problems, and the items are the answers to the questions comprising the test [18].

In this paper we propose to extend certain graphical exploratory techniques and apply them for analyzing the latent data in educational questionnaires. We intend to summarize the values of all the hidden variables and to project the data in a map, where each individual will be placed depending on his/her knowledge profile, allowing the examiner to identify groups with similar background problems, segment heterogeneous groups and perceive the evolution of the learning skills or the abilities acquired during the course. This is not, strictly speaking, a new idea. Statistical methods and other intelligent techniques for analyzing questionnaires and surveys are part of the common knowledge [20]. Moreover, the proliferation of free data mining software has driven many advances in the application of Artificial Intelligence in educational contexts [16], and there exist tools that can generate views of the aforementioned data for easily drawing conclusions and making predictions about the effectivity of a course [14, 19]. On the contrary, the innovation of our approach is not in the use of graphical techniques but in their generalization to sets of items that can be incomplete or imprecise, proposing a novel solution for two frequent problems: (a) that the individual leaves unanswered questions (blank items) and (b) the dispersion of the values of the items associated to the same latent variable is too high thus the average value of the items is no longer a good estimator. According to our approach, a missing or unknown answer in the survey will be represented by an interval or a fuzzy set. For instance, if an item is a number between 0 and 10, an unanswered question will be associated with the interval [0,10]. We will not try to make up a coherent answer for blank items [8], but we will carry the imprecision in all the calculations. In turn, multi-item values will also be represented by intervals or fuzzy sets. For instance, let $(6, 1, 5)$ be the values of three items. With our methodology, instead of replacing this triplet by its mean, the value "4", we could say that the answer is an unknown number in the range $[1, 6]$ (the minimum and the maximum of the answers) or else a fuzzy set, understood as a nested family of intervals at different confidence levels [5].

Using intervals or fuzzy sets for representing unknown values causes that each individual in the map is no longer a point but a figure, whose shape and size determine the coherence and completeness of the answers and whose relative position determines the similarities between it and the other individuals. In this paper we will explain how this map can be generated with the help of interval (or fuzzy) valued fitness function-driven genetic algorithms. Observe that certain modern approaches like Independent Component Analysis (ICA) and Self Organized Maps (SOM) have fuzzy extensions [2, 7], that might also seem appropriate for this problem, however the algorithms we are aware of are not designed for using fuzzy data but for improving the robustness when working with crisp data and thus are intended for solving problems fundamentally different than this. Other nonlinear extensions of PCA, like Curvilinear Component Analysis (CCA) [11] have not yet been extended to the fuzzy case. Indeed, all of these advanced nonlinear techniques are closely related to a technique widely used in psychology: Multi-Dimensional Scaling (MDS) [10]. This last technique has been recently generalized to fuzzy data [6]. The algorithm in this last reference shares a common background with our own extension, and we will compare both in the following sections.

The structure of this paper is as follows: in Section 2 we describe the representation of the information contained in those questionnaires (educational knowledge surveys and tests for diagnosing dyslexia) that will be used in this study. In Section 3 we introduce the concept of Evolutionary Graphical Exploratory Analysis for vague data and discuss its relation with the mentioned questionnaires. In Section 4 we show the outcome of this method in real-world cases. We provide some concluding remarks in Section 5.

2. Questionnaires and representation issues

Two different kinds of questionnaires will be used for assessing students' capacities and detecting learning disorders, respectively. On the one hand, educational knowledge surveys are intended to measure the capability of the student for understanding and solving those problems related to the learning outcomes of a course. These surveys comprise short questions related to specific aspects of these outcomes, and are designed by the teacher. The students can answer by writing a single line, or choosing between several alternatives in a printed or web-based questionnaire.

On the other hand, those tests used for diagnosing a learning disorder are based on different variables, related to the acquisition of language skills, attention deficit, hyperactivity, and other indicators. These last tests are standardized (see Table I) and do not comprise questions but consist of graphical exercises involving shapes, colors and lists of names or numbers.

In either case, the variables of interest are latent, and sets of items or *constructs* must be produced to measure each domain of meaning. In this section we revise the properties of the questionnaires that are used in this study, and define a common

Category	Test	Description
Verbal comprehension	BAPAE	Vocabulary
	BADIG	Verbal orders
	BOEHM	Basic concepts
Logic reasoning	RAVEN	Color
	BADIG	Visual memory
Sensory-motor skills	BENDER	Visual-motor coordination
	BADIG	Perception of shapes
	BAPAE	Spatial relations, Shapes, Orientation
	STAMBACK	Auditive perception, Rhythm
	HARRIS/HPL	Laterality, Pronunciation
	GOODENOUGHT	Spatial orientation, Body scheme
Reading-Writing	TALE	Analysis of reading and writing

Tab. I *Categories of the tests currently applied in Spanish schools for detecting dyslexia in children between 5 and 8 years. The names of the tests are standardized in Spain, see [15] for the bibliographic references.*

representation for both that can be combined with the Evolutionary Graphical Exploratory Analysis described in Section 3.

2.1 Educational Knowledge Surveys

These surveys can be used for assessing the quality of the learning, and they are also meaningful from a didactical point of view, as they allow students to perceive the whole content of the course. Teachers can use these surveys for deciding the best starting level for the lectures [13], specially in Master or pre-doctoral lectures, where the profiles of the students attending the same course are different. Recently this has also been applied to teacher education and certification [21]. When the survey is done at the end of the course, the effectivity of the teaching methodology along with the attitude and dedication of the students is measured. There is certain consensus in the literature about the weak relationship between methodology/dedication and scoring [4]. Because of this, a survey (different than an exam, designed to score the students) is needed.

The design of the constructs involved in a knowledge survey if often guided by the Bloom taxonomy [1, 3]. Other researchers propose taxonomies that classify learning phases [9] that could be useful to design questions that reveal where the student is in the learning curve or assess the critical thinking levels in a given area of the subject. Lastly, with regard to the measurement scales, the constructs in this particular work have been measured by several five-point Likert scales, and therefore the data comprises multi-item values. Each multi-item value is converted into an interval or a fuzzy set by mean of the procedure explained later in this section.

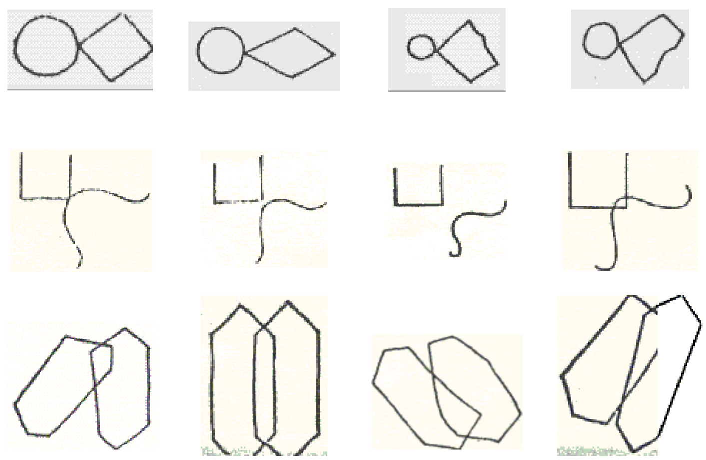

Fig. 1 *Example of some of Bender's tests for detecting dyslexia. Upper part: The angles of the shape in the right are qualified by a list of adjectives that can contain the words "right," "incoherent," "acceptable," "regular" and "extra." Middle and lower part: The relative position between the figures can be "right and separated," "right and touching," "intersecting", etc.*

2.2 Tests for diagnosing dyslexia

All the tests that have been used in this research are currently being used in Spanish schools for detecting dyslexia (see Table I). In Figure 1 we have reproduced one of the graphical exercises involved in the analysis, that is copying some geometric drawings. A psychologist or specialist dyslexia teacher scores these exercises. In this particular case, this expert has to decide whether the angles, relative position and other geometrical properties have been accurately copied or not, choosing between a given set of adjectives such as "right", "incoherent", "acceptable", "regular" or "extra". Other exercises have numerical scores, and generally speaking the data consists of multi-item values, as in the preceding case. However, in this work we have also allowed the expert to express indifference between different responses by means of intervals, as in "lower than 3" or "between acceptable and regular", thus each item may also be an interval. There are 13 categories of tests, that expand to a total of 413 numerical, categorical and interval-valued variables. By aggregating the answers to each one of these categories, we will represent each individual by means of a vector of 13 multi-item variables.

2.3 Fuzzy representation of multi-item values

For estimating the values of the latent variables, multi-item variables are aggregated. Following previous works [18], we have decided that converting the aggregate into a number is not always convenient, because relevant information is lost. Therefore a set-valued aggregation operator is used instead. In this work we will represent each multi-item value by a fuzzy set (or, as a limit case, an interval) such that its α-cuts are confidence intervals with degree $1-\alpha$ of the mean value of the latent variable. This procedure converts a questionnaire into a vector of fuzzy values (or intervals), one for each variable of interest, and this transformation loses less information than aggregating the items with central tendency measures. The numerical algorithm is described in [18], and it is illustrated in the example that follows, condensed from that reference.

Example 1 *Let us suppose that a latent variable x_0 is described by the following set of items:*

$$X = (2, 1, 3, 3, 2, 2, 4). \tag{1}$$

We will assume that X is a simple random sample of a population whose mean is the unknown value x_0. Let $\widetilde{A}$ be the membership function of the fuzzy set that describes our knowledge about x_0. Then, the family of its cuts $\{\widetilde{A}^\alpha\}$ is a nested family of confidence intervals such that $P(x_0 \in A^\alpha) = P(A^\alpha) \geq 1-\alpha$. The membership function can be built from the quantiles of the bootstrap distribution of the sample mean, as shown in Figure 2.

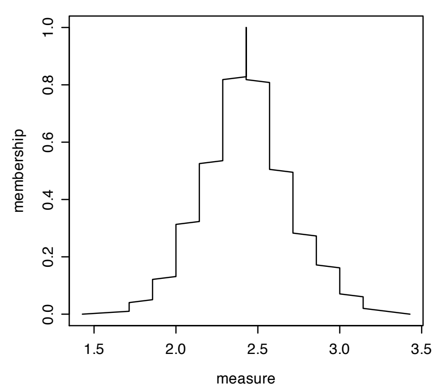

Fig. 2 *Bootstrap-based fuzzy representation of the multi-item value in Example 1.*

Observe also that this construction can easily accommodate interval-valued items, using interval arithmetic for computing the sample mean and the lower and upper bounds of the quantiles of the bootstrap distribution. Finally, it is remarked that, as a particular case, a missing item can be represented by an interval spanning the whole domain of the variable, thus this representation is intrinsically able to handle missing data.

3. Evolutionary Graphical Exploratory Analysis

There are many different techniques for performing graphical exploratory analysis of data, as mentioned in the introduction: Sammon maps, Principal Component

Analysis (PCA), Multidimensional Scaling (MDS), self-organized maps (SOM), etc. [6]. These methods project the instances as points in a low dimensional Euclidean space so that their proximity reflects the similarity of their variables. However, we have mentioned that the surveys can be incomplete or contain conflicting answers. Summing up, an incomplete survey is taken as the set of all surveys with any valid value in place of the missing answer. A multi-valued item can also be understood as a set, as we shown in the preceding section. The most immediate consequence of this representation is that the projection of an instance is no longer a point, but a shape whose size will be larger as the more incomplete or imprecise the information about the individual is.

In this section we will describe first the theoretical basis of our generalization of the MDS algorithm to fuzzy data, paying special attention to the differences between our algorithm and its closest precedent in the literature [6]. Our definition of the stress function and the freedom given to the shape of the projections prevent the use of classical optimization techniques, as done in the mentioned reference, thus we make use of a nonstandard Genetic Algorithm, described first in [17]. The main properties of this algorithm are also described in this section.

3.1 Fuzzy MDS

This extension from a map of points to a map of shapes has already been done for some of the techniques mentioned before. For instance, Fuzzy MDS, as described in [6], extends MDS to the case where the distance matrix comprises intervals or fuzzy numbers, as happens in our problem. Crisp MDS consists in finding a low-dimensional cloud of points that minimizes an stress function. That function measures the difference between the matrix of distances among the data and the matrix of distances among the elements of this last cloud. The interval (or fuzzy) extension of this algorithm defines an interval (fuzzy) valued stress function that bounds the difference between the imprecisely known matrix of distances between the objects and the interval (fuzzy) valued distance matrix between a set of shapes in the low-dimensional projection.

Let us assume for the time being that the distance between two surveys is an interval (the extension to the fuzzy case is straightforward, since it suffices to apply the following to each cut of the fuzzy distance). For two imprecisely measured multivariate values $x_i = [x_{i1}^-, x_{i1}^+] \times \ldots \times [x_{if}^-, x_{if}^+]$ and $x_j = [x_{j1}^-, x_{j1}^+] \times \ldots \times [x_{jf}^-, x_{jf}^+]$, with f features each, the set of distances between their possible values is the interval

$$D_{ij} = \left\{ \sqrt{\textstyle\sum_{k=1}^{f} (x_{ik} - x_{jk})^2} \mid x_{ik} \in [x_{ik}^-, x_{ik}^+], x_{jk} \in [x_{jk}^-, x_{jk}^+], 1 \le k \le f \right\}. \tag{2}$$

Some authors have used a distance similar to this before [6], however they further assumed that the shape of projection of an imprecise case is always a circle. We have found that, in our problem, this is a too restrictive hypothesis. Instead, we propose to approximate the shape of the projections by a polygon (see Figure 3, left part) whose radii R_{ij}^+ and R_{ij}^- are not free variables, but depend on the distances between the cases.

For a multivariate sample of imprecise data $(x_1, \ldots, x_N)$, let $\overline{x}_i$ be the crisp centerpoint of the imprecise value x_i (the center of gravity, if an interval, or the

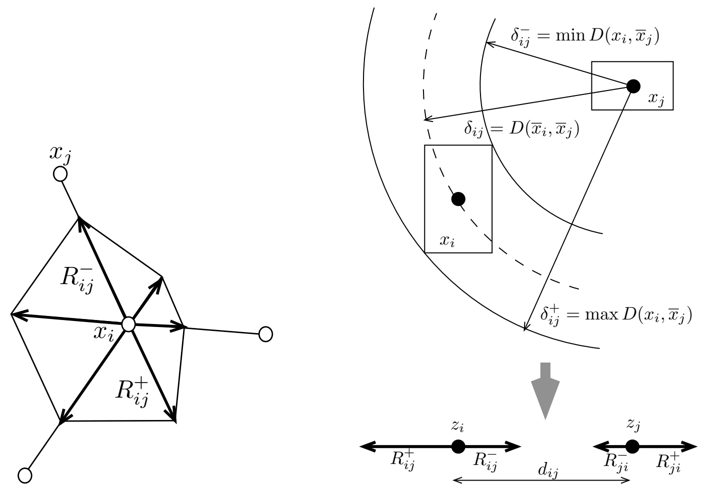

Fig. 3 *Left part: The projected data are polygons defined by the distances R_{ij} in the directions that pairwise join the examples. Right part: The distance between the projections of x_i and x_j is between $d_{ij} - R_{ij}^- - R_{ji}^-$ and $d_{ij} + R_{ij}^+ - R_{ji}^+$*

modal point, if fuzzy), and let $((z_{11}, \ldots, z_{1r}), \ldots, (z_{N1}, \ldots, z_{Nr}))$ be a crisp projection, with dimension r, of that set. We propose that the radii R_{ij}^+ and R_{ij}^- depend on the distance between x_i and $\overline{x}_j$ (see the right part of Figure 3 for a graphical explanation) as follows

$$R_{ij}^+ = d_{ij}\left(\frac{\delta_{ij}^+}{\delta_{ij}} - 1\right) \quad R_{ij}^- = d_{ij}\left(\frac{\delta_{ij}}{\delta_{ij}^-} - 1\right) \tag{3}$$

where $d_{ij} = \sqrt{\sum_{k=1}^{r}(z_{ik} - z_{jk})^2}$, $\delta_{ij} = \{D(\overline{x}_i, \overline{x}_j)\}$, $\delta_{ij}^+ = \max\{D(x_i, \overline{x}_j)\}$, and $\delta_{ij}^- = \min\{D(x_i, \overline{x}_j)\}$. We also propose that the value of the stress function our map has to minimize is

$$\sum_{i=1}^{N}\sum_{j=i+1}^{N} d_H(D_{ij}, [d_{ij} - R_{ij}^- - R_{ji}^-, d_{ij} + R_{ij}^+ + R_{ji}]^+)^2 \tag{4}$$

where d_H is the Haussdorff distance between intervals.

3.2 Characteristic points

For gaining insight into the actual values of the spatial coordinates of the elements displayed in the map, we propose to add several prototypal, fictitious sets of items (we will call them "characteristic points") corresponding to a test without mistakes, a test which is completely wrong, one section well answered but the remaining ones wrong, etc. In the final map, these points will be approximately placed in a circle enclosing the projections of the individuals. With the help of these points, the map can be used for evaluating the capacities of a student or diagnosing a disorder by comparing it with its closest characteristic point.

3.3 Evolutionary algorithm

An evolutionary algorithm is used for searching the map optimizing the stress function (4). In previous works we have shown that interval and fuzzy fitness functions can be optimized by certain extensions of multiobjective genetic algorithms. In this paper we have used the extended NGSA-II defined in [17], whose main components are summarized in the following paragraphs.

- **Representation:** Since the shape of each element in the map can be computed given the centerpoints of both the sets and the elements of the map, as described in Section 3.1, each map can be univocally determined from a set of coordinates in the plane, thus each chromosome consists of the concatenation of so many pairs of numbers as individuals, plus one pair for each characteristic point (i.e. "Everything", "Nothing", "Only Subject X", "Every Subject but X", etc). The chromosome is fixed-length, and real coding is used.

- **Objective Function:** The genetic algorithm must minimize the expression defined in eq. (4). However, observe that this equation does not evaluate to a number, but to an interval or a fuzzy set. Generally speaking, one cannot properly define a total order between interval or fuzzy sets, and therefore the concept of "minimum" must be replaced by that of "set of minimal elements", which is closely related to the definition of a Pareto front in multicriteria optimization [22]. There exist, however, many different proposals for precedence operators or rankings between fuzzy sets, some of which could be used to define a total order over the solutions and be combined with a suitable scalar evolutionary search [12]. In this work, however, the precedence operator induces a partial order in the set of solutions, thus the search will produce a set of nondominated maps.

- **Evolutionary Scheme:** A generational approach with the multiobjective NSGA-II replacement strategy is considered. Binary tournament selection based on the crowding distance in the objective function space is used. The precedence operator derives from the bayesian coherent inference with an imprecise prior, the dominated sorting is based on the product of the lower probabilities of precedence, and the crowding in based on the Hausdorff distance, as described in [17].

- **Genetic Operators:** Arithmetic crossover is used for combining two chains. The mutation operator consists in performing crossover with a randomly generated chain.

4. Results

In this section we will illustrate, with the help of real-world datasets, how to process different kinds of tests and interpret the resulting maps. First, it will be shown how to segment heterogeneous groups of students and how to study the temporal evolution of the learning, which will be represented by arrows. This is useful for finding groups of students that cannot follow the course timeline or those concepts

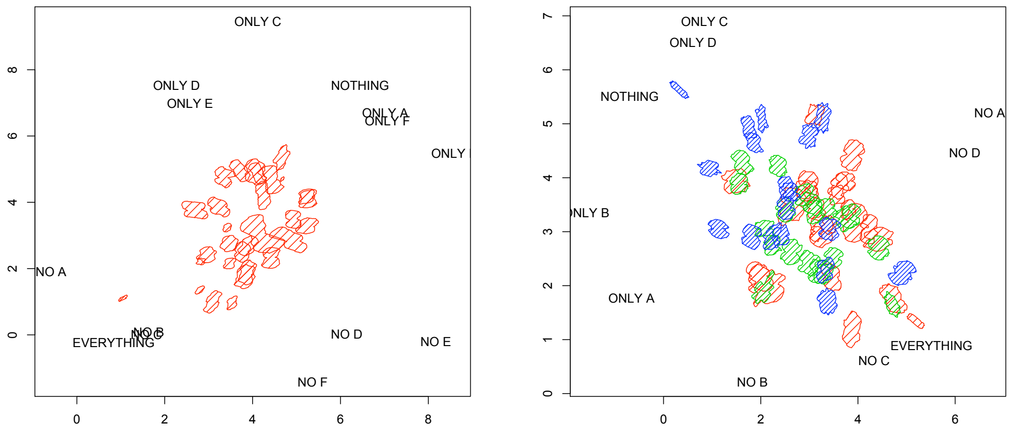

Fig. 4 *Left part: Differences in knowledge of Statistics for students in Ingenieria Telematica. Right part: Differences in knowledge about Computer Science between the students of Ingenieria Tecnica Industrial specialized in Chemistry, Electricity and Mechanics.*

that are learned faster by each group of students. Lastly, we will apply the same techniques to a group of preschoolers with learning disorders and use the techniques developed in this research for analyzing their evolution.

4.1 Knowledge Surveys I: Variation of individual capacities in the same group and between groups

In the left part of Figure 4 a diagram for 30 students of subject "Statistics" in *Ingenieria Telematica* at Oviedo University, is shown. The data was acquired at the beginning of the course 2009-2010. This survey is related to students' prerrequisite skills in Algebra (A), Logic (B), Electronics (C), Numerical Analysis (D), Probability (E) and Physics (F). The positions of the characteristic points have been marked with labels. Those points are of the type "A" (all the questions about the subject "A" are correct, the others are erroneous) "NO A" (all the questions except "A" ones are correct, the opposite situation), etc.

In the right part of Figure 4 we have plotted together the results of three different groups, who attend lectures by the same teacher. Each intensification has been coded with a distinctive colour. This teacher has evaluated, as before, the initial knowledge of the students in subjects that are a prerequisite. From the graphic in that figure, the most relevant fact is that the students of the intensification coded in red *(Ingenieria Industrial)* consider themselves better prepared than those coded in blue *(Ingeniería Tecnica Industrial Electrica)*, with the green group in an intermediate position, closer to red *(Ingeniería Tecnica Industrial Quimica)*. All the students of all the groups have a neutral orientation to math subjects, and some

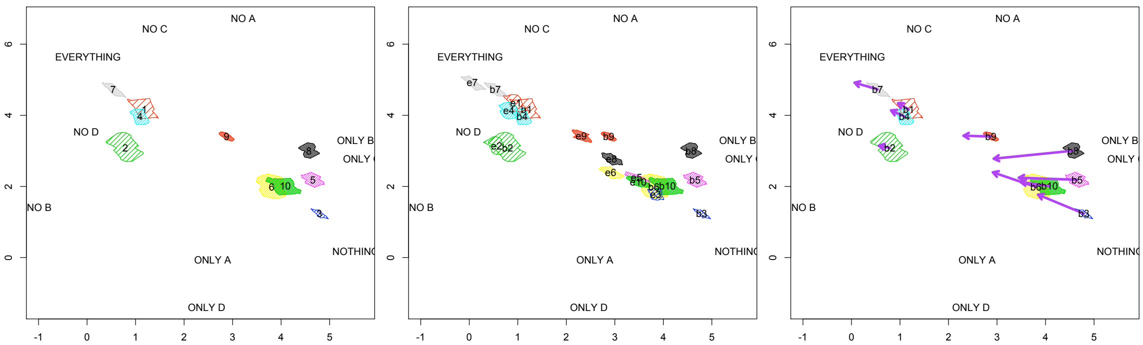

Fig. 5 *Evolution of the learning of pre-doctoral students. Left part: Initial survey. Center: superposition of initial and final maps. Right part: The displacement has been shown by arrows.*

students in the blue group think that their background is adequate only in subjects C (Operating Systems) and D (Internet).

4.2 Knowledge Surveys II: Evolution of learning capabilities

Ten pre-doctoral students in Computer Science, Physics and Mathematics attending a research master were analyzed. The background of these students is heterogeneous. In the survey the students were asked 36 questions about the variables "Control Algorithms" (A), "Statistical Data Analysis" (B), "Numerical Algorithms" (C) and "Lineal Models" (D). The left part of Figure 5 shows that there is a large dispersion between the initial knowledges. Since the course had strong theoretic foundations, students from technical degrees like Computer Science evaluated themselves with the lowest scores (shapes in the right part of each figure).

The same survey, repeated at the end of the course, shows that all the students moved to the left, closer to characteristic point "EVERYTHING". Additionally, the displacement has been larger for the students in the group at the right. This displacement can be seen clearly in the right part of the same figure, where the shapes obtained from the final survey were replaced by arrows that begin in the initial position and end in the final center. The length of the arrows is related with the progress of the student during the course, showing that those students that scored the highest marks in the course (those who also considered themselves best prepared at the beginning of the course) did not make a good use of the course, which on the contrary was able to improve the capabilities of students in technical degrees.

4.3 Diagnosis of dyslexia

For this last experiment we have collected a sample of 65 infants between 5 and 6 years old, in urban schools of Asturias (Spain). Afterwards, the same children were

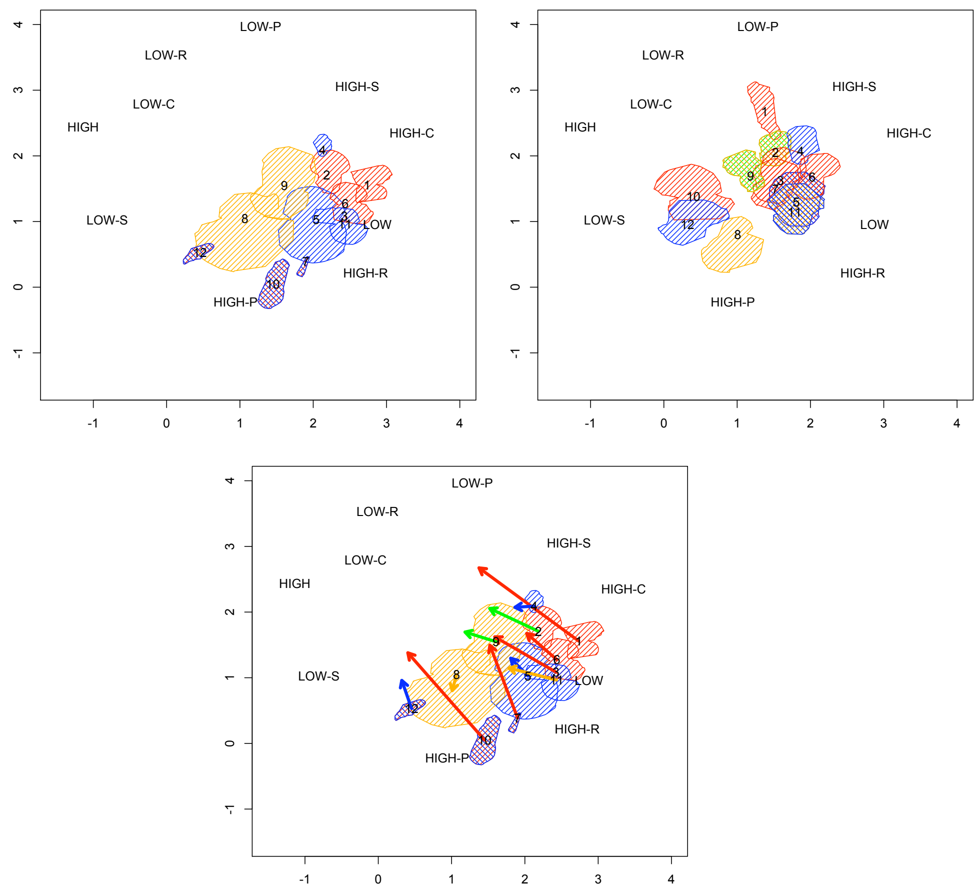

Fig. 6 *Evolution of dyslexia. Left part: 4-5 years. Center: 6-7 years. Right part: The displacement has been shown by arrows.*

examined by a psychologist, who assigned each one of them a color: green (normal child), red (dyslexic), orange (slight dyslexia) and blue (attention disorder). In some cases the child was too young for a definite diagnostic and the expert assigned two colors to them (for example, red plus blue mean "might be dyslexia or an attention disorder"). We selected twelve children with potential problems and repeated the tests one year later. We have included the characteristic points of four latent variables: Reasoning (R), Visual-Motor coordination (C), Shape Perception (P) and Spatial Orientation (S).

In the upper-left part of Figure 6 the initial map of the children is shown.

Observe that the red shapes (children suffering dislexia) are concentrated on the right part of the map (low scores in all of the latent variables, as indicated by the axis joining the characteristic points "LOW" and "HIGH") and also tend to be in the upper part of the map (lower values in Shape Perception, measured by Bender's tests). The size of some shapes reflects that there are a moderate amount of missing values, however the map shows that the values of the missing items are not too relevant in this stage of the diagnosis, since the intersection of shapes with different colors is low (except for individual number 5, which incidentally had his diagnosis revised one year later). Observe also that the three individuals labelled "red + blue" are clearly positioned nearer to the area of attention disorder, and this may indicate that the expert could have used this map for gaining insight in her diagnosis.

The upper-right part of the same figure illustrate the map of these children, one year later. As expected, the skills of all individuals have been enhanced, and all of them are nearer to the characteristic point "HIGH", which is a positive result. This is clearly seen in the lower part of the figure, where both maps are superimposed and the shapes corresponding to the latest test have been removed and replaced by an arrow joining the centers of the initial and final shapes. The color of the shape reflects the final diagnosis of the expert in this second test. Observe that children 10 and 12 apparently have evolved to the same area of the map, but the expert has labelled the individual number 10 as "dyslexic". In this case, there is a significant overlap between the shapes of these two individuals and the information given by the tests is too incomplete for being reliable. This last sanity check would have not been possible without the extra information given by the size and shape of the projection that are provided by this method.

5. Conclusions

In this work we have extended the Multidimensional Scaling to imprecise data, and exploited the new capabilities of the algorithm for producing a method able to process incomplete or non consistent tests measuring learning capabilities and learning disorders. The map of a group of individuals comprises several shapes, whose volumes measure the degree to which a survey has missing data and whose relative positions depend on the similarities between individuals. We have shown with the help of real-world data that these maps can help detecting heterogeneous groups and measuring the capabilities of the student after the course, and can also be used during the diagnosis of certain learning disorders, being able to condense large amounts of data in a simple graph that permits gaining insight in the evolution of a group of individuals, even when the available data is incomplete or imprecise.

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