Data Mining and Knowledge Discovery, 8, 53–87, 2004
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Mining Frequent Patterns without Candidate Generation: A Frequent-Pattern Tree Approach*

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Editor: Heikki Mannila

Received May 21, 2000; Revised April 21, 2001

Abstract. Mining frequent patterns in transaction databases, time-series databases, and many other kinds of databases has been studied popularly in data mining research. Most of the previous studies adopt an *Apriori*-like candidate set generation-and-test approach. However, candidate set generation is still costly, especially when there exist a large number of patterns and/or long patterns.

In this study, we propose a novel frequent-pattern tree (FP-tree) structure, which is an extended prefix-tree structure for storing compressed, crucial information about frequent patterns, and develop an efficient FP-tree-based mining method, *FP-growth*, for mining *the complete set of frequent patterns* by pattern fragment growth. Efficiency of mining is achieved with three techniques: (1) a large database is compressed into a condensed, smaller data structure, FP-tree which avoids costly, repeated database scans, (2) our FP-tree-based mining adopts a pattern-fragment growth method to avoid the costly generation of a large number of candidate sets, and (3) a partitioning-based, divide-and-conquer method is used to decompose the mining task into a set of smaller tasks for mining confined patterns in conditional databases, which dramatically reduces the search space. Our performance study shows that the *FP-growth* method is efficient and scalable for mining both long and short frequent patterns, and is about an order of magnitude faster than the *Apriori* algorithm and also faster than some recently reported new frequent-pattern mining methods.

Keywords: frequent pattern mining, association mining, algorithm, performance improvements, data structure

*The work was done at Simon Fraser University, Canada, and it was supported in part by the Natural Sciences and Engineering Research Council of Canada, and the Networks of Centres of Excellence of Canada.

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1. Introduction

Frequent-pattern mining plays an essential role in mining associations (Agrawal et al., 1993, 1996; Agrawal and Srikant, 1994; Mannila et al., 1994), correlations (Brin et al., 1997), causality (Silverstein et al., 1998), sequential patterns (Agrawal and Srikant, 1995), episodes (Mannila et al., 1997), multi-dimensional patterns (Lent et al., 1997; Kamber et al., 1997), max-patterns (Bayardo, 1998), partial periodicity (Han et al., 1999), emerging patterns (Dong and Li, 1999), and many other important data mining tasks.

Most of the previous studies, such as Agrawal and Srikant (1994), Mannila et al. (1994), Agrawal et al. (1996), Savasere et al. (1995), Park et al. (1995), Lent et al. (1997), Sarawagi et al. (1998), Srikant et al. (1997), Ng et al. (1998) and Grahne et al. (2000), adopt an *Apriori*-like approach, which is based on the *anti-monotone Apriori heuristic* (Agrawal and Srikant, 1994): *if any length k pattern is not frequent in the database, its length* $(k + 1)$ *super-pattern can never be frequent*. The essential idea is to iteratively generate the set of candidate patterns of length $(k+1)$ from the set of frequent-patterns of length k (for $k \geq 1$), and check their corresponding occurrence frequencies in the database.

The *Apriori* heuristic achieves good performance gained by (possibly significantly) reducing the size of candidate sets. However, in situations with a large number of frequent patterns, long patterns, or quite low minimum support thresholds, an *Apriori*-like algorithm may suffer from the following two nontrivial costs:

– It is costly to handle a huge number of candidate sets. For example, if there are 10^4 frequent 1-itemsets, the *Apriori* algorithm will need to generate more than 10^7 length-2 candidates and accumulate and test their occurrence frequencies. Moreover, to discover a frequent pattern of size 100, such as $\{a_1, \ldots, a_{100}\}$, it must generate $2^{100} - 2 \approx 10^{30}$ candidates in total. This is the inherent cost of candidate generation, no matter what implementation technique is applied.
– It is tedious to repeatedly scan the database and check a large set of candidates by pattern matching, which is especially true for mining long patterns.

Can one develop a method that may avoid candidate generation-and-test and utilize some novel data structures to reduce the cost in frequent-pattern mining? This is the motivation of this study.

In this paper, we develop and integrate the following three techniques in order to solve this problem.

First, a novel, compact data structure, called *frequent-pattern tree*, or FP-tree in short, is constructed, which is an extended prefix-tree structure storing crucial, quantitative information about frequent patterns. To ensure that the tree structure is compact and informative, only frequent length-1 items will have nodes in the tree, and the tree nodes are arranged in such a way that more frequently occurring nodes will have better chances of node sharing than less frequently occurring ones. Our experiments show that such a tree is compact, and it is sometimes orders of magnitude smaller than the original database. Subsequent frequent-pattern mining will only need to work on the FP-tree instead of the whole data set.

Second, an FP-tree-based pattern-fragment growth mining method is developed, which starts from a frequent length-1 pattern (as an initial *suffix pattern*), examines only its *conditional-pattern base* (a "sub-database" which consists of the set of frequent items co-occurring with the suffix pattern), constructs its (*conditional*) FP-tree, and performs mining recursively with such a tree. The pattern growth is achieved via concatenation of the suffix pattern with the new ones generated from a conditional FP-tree. Since the frequent itemset in any transaction is always encoded in the corresponding path of the frequent-pattern trees, pattern growth ensures the completeness of the result. In this context, our method is not *Apriori*-like *restricted generation-and-test* but *restricted test only*. The major operations of mining are count accumulation and prefix path count adjustment, which are usually much less costly than candidate generation and pattern matching operations performed in most *Apriori*-like algorithms.

Third, the search technique employed in mining is a *partitioning-based*, *divide-and-conquer method* rather than *Apriori*-like *level-wise generation of the combinations of frequent itemsets*. This dramatically reduces the size of *conditional-pattern base* generated at the subsequent level of search as well as the size of its corresponding *conditional FP-tree*. Moreover, it transforms the problem of finding long frequent patterns to looking for shorter ones and then concatenating the suffix. It employs the least frequent items as suffix, which offers good selectivity. All these techniques contribute to substantial reduction of search costs.

A performance study has been conducted to compare the performance of *FP-growth* with two representative frequent-pattern mining methods, *Apriori* (Agrawal and Srikant, 1994) and *TreeProjection* (Agarwal et al., 2001). Our study shows that *FP-growth* is about an order of magnitude faster than *Apriori*, especially when the data set is dense (containing many patterns) and/or when the frequent patterns are long; also, *FP-growth* outperforms the *TreeProjection* algorithm. Moreover, our FP-tree-based mining method has been implemented in the DBMiner system and tested in large transaction databases in industrial applications.

Although *FP-growth* was first proposed briefly in Han et al. (2000), this paper makes additional progress as follows.

- The properties of FP-tree are thoroughly studied. Also, we point out the fact that, although it is often compact, FP-tree may not always be minimal.
- Some optimizations are proposed to speed up *FP-growth*, for example, in Section 3.2, a technique to handle single path FP-tree has been further developed for performance improvements.
- A database projection method has been developed in Section 4 to cope with the situation when an FP-tree cannot be held in main memory—the case that may happen in a very large database.
- Extensive experimental results have been reported. We examine the size of FP-tree as well as the turning point of *FP-growth* on data projection to building FP-tree. We also test the fully integrated *FP-growth* method on large datasets which cannot fit in main memory.

The remainder of the paper is organized as follows. Section 2 introduces the FP-tree structure and its construction method. Section 3 develops an FP-tree-based frequent-pattern mining algorithm, *FP-growth*. Section 4 explores techniques for scaling *FP-growth* in large databases. Section 5 presents our performance study. Section 6 discusses the issues on further improvements of the method. Section 7 summarizes our study and points out some future research issues.

2. Frequent-pattern tree: Design and construction

Let $I = \{a_1, a_2, \ldots, a_m\}$ be a set of items, and a *transaction database* $DB = \langle T_1, T_2, \ldots, T_n \rangle$, where T_i ($i \in [1 \ldots n]$) is a transaction which contains a set of items in I. The *support*[1] (or occurrence frequency) of a *pattern* A, where A is a set of items, is the number of transactions containing A in *DB*. A pattern A is *frequent* if A's support is no less than a predefined *minimum support threshold*, ξ.

Given a transaction database *DB* and a minimum support threshold ξ, the problem of *finding the complete set of frequent patterns* is called the frequent-pattern mining problem.

2.1. Frequent-pattern tree

To design a compact data structure for efficient frequent-pattern mining, let's first examine an example.

Example 1. Let the transaction database, *DB*, be the first two columns of Table 1, and the minimum support threshold be 3 (i.e., $\xi = 3$).

A compact data structure can be designed based on the following observations:

1. Since only the frequent items will play a role in the frequent-pattern mining, it is necessary to perform one scan of transaction database *DB* to identify the set of frequent items (with *frequency count* obtained as a by-product).
2. If the *set* of frequent items of each transaction can be stored in some compact structure, it may be possible to avoid repeatedly scanning the original transaction database.
3. If multiple transactions share a set of frequent items, it may be possible to merge the shared sets with the number of occurrences registered as *count*. It is easy to check whether two sets are identical if the frequent items in all of the transactions are listed according to a fixed order.

Table 1. A transaction database as running example.

TID	Items bought	(Ordered) frequent items
100	f, a, c, d, g, i, m, p	f, c, a, m, p
200	a, b, c, f, l, m, o	f, c, a, b, m
300	b, f, h, j, o	f, b
400	b, c, k, s, p	c, b, p
500	a, f, c, e, l, p, m, n	f, c, a, m, p

4. If two transactions share a common prefix, according to some sorted order of frequent items, the shared parts can be merged using one prefix structure as long as the *count* is registered properly. If the frequent items are sorted in their *frequency descending order*, there are better chances that more prefix strings can be shared.

With the above observations, one may construct a frequent-pattern tree as follows.

First, a scan of *DB* derives a *list* of frequent items, $\langle(f{:}4), (c{:}4), (a{:}3), (b{:}3), (m{:}3), (p{:}3)\rangle$ (the number after ":" indicates the support), in which items are ordered in frequency-descending order. This ordering is important since each path of a tree will follow this order. For convenience of later discussions, the frequent items in each transaction are listed in this ordering in the rightmost column of Table 1.

Second, the root of a tree is created and labeled with "*null*". The FP-tree is constructed as follows by scanning the transaction database *DB* the second time.

1. The scan of the first transaction leads to the construction of the first branch of the tree: $\langle(f{:}1), (c{:}1), (a{:}1), (m{:}1), (p{:}1)\rangle$. Notice that the frequent items in the transaction are listed according to the order in the *list* of frequent items.
2. For the second transaction, since its (ordered) frequent item list $\langle f, c, a, b, m\rangle$ shares a common prefix $\langle f, c, a\rangle$ with the existing path $\langle f, c, a, m, p\rangle$, the count of each node along the prefix is incremented by 1, and one new node $(b{:}1)$ is created and linked as a child of $(a{:}2)$ and another new node $(m{:}1)$ is created and linked as the child of $(b{:}1)$.
3. For the third transaction, since its frequent item list $\langle f, b\rangle$ shares only the node $\langle f\rangle$ with the f-prefix subtree, f's count is incremented by 1, and a new node $(b{:}1)$ is created and linked as a child of $(f{:}3)$.
4. The scan of the fourth transaction leads to the construction of the second branch of the tree, $\langle(c{:}1), (b{:}1), (p{:}1)\rangle$.
5. For the last transaction, since its frequent item list $\langle f, c, a, m, p\rangle$ is identical to the first one, the path is shared with the count of each node along the path incremented by 1.

To facilitate tree traversal, an item header table is built in which each item points to its first occurrence in the tree via a node-link. Nodes with the same item-name are linked in sequence via such *node-links*. After scanning all the transactions, the tree, together with the associated node-links, are shown infigure 1.

Based on this example, a *frequent-pattern tree* can be designed as follows.

Definition 1 (*FP-tree*). A *frequent-pattern tree* (or *FP-tree* in short) is a tree structure defined below.

1. It consists of one root labeled as "*null*", a set of item-prefix subtrees as the children of the root, and a frequent-item-header table.
2. Each node in the item-prefix subtree consists of three fields: *item-name*, *count*, and *node-link*, where *item-name* registers which item this node represents, *count* registers the number of transactions represented by the portion of the path reaching this node, and

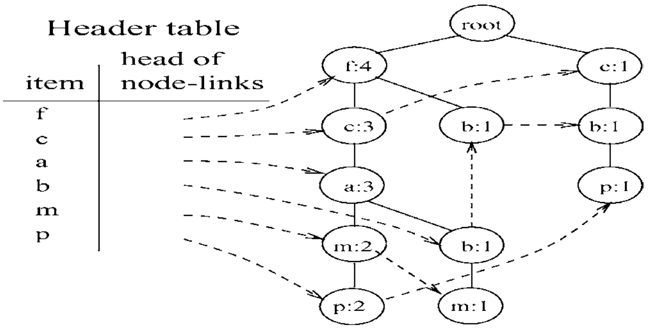


Figure 1. The FP-tree in Example 1.

node-link links to the next node in the FP-tree carrying the same item-name, or null if there is none.

3. Each entry in the frequent-item-header table consists of two fields, (1) *item-name* and (2) *head of node-link* (a pointer pointing to the first node in the FP-tree carrying the *item-name*).

Based on this definition, we have the following FP-tree construction algorithm.

Algorithm 1 (FP-tree construction)**.**

Input: A transaction database *DB* and a minimum support threshold ξ.
Output: FP-tree, the frequent-pattern tree of *DB*.
Method: The FP-tree is constructed as follows.

1. Scan the transaction database *DB* once. Collect F, the set of frequent items, and the support of each frequent item. Sort F in support-descending order as *FList*, the *list* of frequent items.
2. Create the root of an FP-tree, T, and label it as "null". For each transaction *Trans* in *DB* do the following.

   Select the frequent items in *Trans* and sort them according to the order of *FList*. Let the sorted frequent-item list in *Trans* be $[p \mid P]$, where p is the first element and P is the remaining list. Call *insert_tree*$([p \mid P], T)$.

   The function *insert_tree*$([p \mid P], T)$ is performed as follows. If T has a child N such that $N.\textit{item-name} = p.\textit{item-name}$, then increment N's count by 1; else create a new node N, with its count initialized to 1, its parent link linked to T, and its node-link linked to the nodes with the same *item-name* via the node-link structure. If P is nonempty, call *insert_tree*(P, N) recursively.

Analysis. The FP-tree construction takes exactly two scans of the transaction database: The first scan collects the set of frequent items, and the second scan constructs the FP-tree. The cost of inserting a transaction *Trans* into the FP-tree is $\mathcal{O}(|freq(Trans)|)$, where $freq(Trans)$ is the set of frequent items in *Trans*. We will show that the FP-tree contains the complete information for frequent-pattern mining.

2.2. *Completeness and compactness of FP-tree*

There are several important properties of FP-tree that can be derived from the FP-tree construction process.

Given a transaction database *DB* and a support threshold ξ. Let F be the frequent items in *DB*. For each transaction T, $freq(T)$ is the set of frequent items in T, i.e., $freq(T) = T \cap F$, and is called the *frequent item projection* of transaction T. According to the *Apriori* principle, the set of frequent item projections of transactions in the database is sufficient for mining the complete set of frequent patterns, because an infrequent item plays no role in frequent patterns.

Lemma 2.1. *Given a transaction database DB and a support threshold ξ, the complete set of frequent item projections of transactions in the database can be derived from DB's FP-tree.*

Rationale. Based on the FP-tree construction process, for each transaction in the *DB*, its frequent item projection is mapped to one path in the FP-tree.

For a path $a_1a_2 \ldots a_k$ from the root to a node in the FP-tree, let c_{a_k} be the count at the node labeled a_k and c'_{a_k} be the sum of counts of children nodes of a_k. Then, according to the construction of the FP-tree, the path registers frequent item projections of $c_{a_k} - c'_{a_k}$ transactions.

Therefore, the FP-tree registers the complete set of frequent item projections without duplication.

Based on this lemma, after an FP-tree for *DB* is constructed, it contains the complete information for mining frequent patterns from the transaction database. Thereafter, only the FP-tree is needed in the remaining mining process, regardless of the number and length of the frequent patterns.

Lemma 2.2. *Given a transaction database DB and a support threshold ξ. Without considering the* (*null*) *root, the size of an FP-tree is bounded by $\sum_{T \in DB} |freq(T)|$, and the height of the tree is bounded by $\max_{T \in DB}\{|freq(T)|\}$, where $freq(T)$ is the frequent item projection of transaction T.*

Rationale. Based on the FP-tree construction process, for any transaction T in *DB*, there exists a path in the FP-tree starting from the corresponding item prefix subtree so that the set of nodes in the path is exactly the same set of frequent items in T. The root is the only extra node that is not created by frequent-item insertion, and each node contains one node-link and one count. Thus we have the bound of the size of the tree stated in the Lemma.

The height of any p-prefix subtree is the maximum number of frequent items in any transaction with p appearing at the head of its frequent item list. Therefore, the height of the tree is bounded by the maximal number of frequent items in any transaction in the database, if we do not consider the additional level added by the root.

Lemma 2.2 shows an important benefit of FP-tree: the size of an FP-tree is bounded by the size of its corresponding database because each transaction will contribute at most one path to the FP-tree, with the length equal to the number of frequent items in that transaction. Since there are often a lot of sharings of frequent items among transactions, the size of the tree is usually much smaller than its original database. Unlike the *Apriori*-like method which may generate an exponential number of candidates in the worst case, under no circumstances, may an FP-tree with an exponential number of nodes be generated.

FP-tree is a highly compact structure which stores the information for frequent-pattern mining. Since a single path "$a_1 \rightarrow a_2 \rightarrow \cdots \rightarrow a_n$" in the a_1-prefix subtree registers all the transactions whose maximal frequent set is in the form of "$a_1 \rightarrow a_2 \rightarrow \cdots \rightarrow a_k$" for any $1 \leq k \leq n$, the size of the FP-tree is substantially smaller than the size of the database and that of the candidate sets generated in the association rule mining.

The items in the frequent item set are ordered in the support-descending order: More frequently occurring items are more likely to be shared and thus they are arranged closer to the top of the FP-tree. This ordering enhances the compactness of the FP-tree structure. However, this does not mean that the tree so constructed *always* achieves the maximal compactness. With the knowledge of particular data characteristics, it is sometimes possible to achieve even better compression than the frequency-descending ordering. Consider the following example. Let the set of transactions be: $\{adef, bdef, cdef, a, a, a, b, b, b, c, c, c\}$, and the minimum support threshold be 3. The frequent item set associated with support count becomes $\{a{:}4, b{:}4, c{:}4, d{:}3, e{:}3, f{:}3\}$. Following the item frequency ordering $a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow f$, the FP-tree constructed will contain 12 nodes, as shown in figure 2(a). However, following another item ordering $f \rightarrow d \rightarrow e \rightarrow a \rightarrow b \rightarrow c$, it will contain only 9 nodes, as shown in figure 2(b).

The compactness of FP-tree is also verified by our experiments. Sometimes a rather small FP-tree is resulted from a quite large database. For example, for the database *Connect-4* used in *MaxMiner* (Bayardo, 1998), which contains 67,557 transactions with 43 items in each transaction, when the support threshold is 50% (which is used in the *MaxMiner* experiments

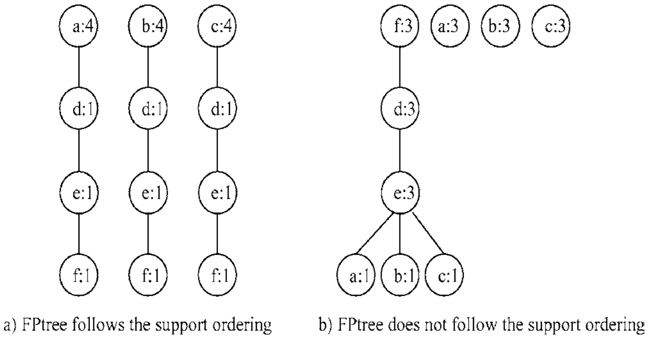

a:4
d:1
e:1
f:1
b:4
d:1
e:1
f:1
c:4
d:1
e:1
f:1
f:3
a:3
b:3
c:3
d:3
e:3
a:1
b:1
c:1
a) FPtree follows the support ordering
b) FPtree does not follow the support ordering

Figure 2. FP-tree constructed based on frequency descending ordering may not always be minimal.

(Bayardo, 1998)), the total number of occurrences of frequent items is 2,219,609, whereas the total number of nodes in the FP-tree is 13,449 which represents a reduction ratio of 165.04, while it still holds hundreds of thousands of frequent patterns! (Notice that for databases with mostly short transactions, the reduction ratio is not that high.) Therefore, it is not surprising some gigabyte transaction database containing many long patterns may even generate an FP-tree that fits in main memory. Nevertheless, one cannot assume that an FP-tree can always fit in main memory no matter how large a database is. Methods for highly scalable *FP-growth* mining will be discussed in Section 5.

3. Mining frequent patterns using FP-tree

Construction of a compact FP-tree ensures that subsequent mining can be performed with a rather compact data structure. However, this does not automatically guarantee that it will be highly efficient since one may still encounter the combinatorial problem of candidate generation if one simply uses this FP-tree to generate and check all the candidate patterns.

In this section, we study how to explore the compact information stored in an FP-tree, develop the principles of frequent-pattern growth by examination of our running example, explore how to perform further optimization when there exists a single prefix path in an FP-tree, and propose a frequent-pattern growth algorithm, *FP-growth*, for mining the *complete set of frequent patterns* using FP-tree.

3.1. Principles of frequent-pattern growth for FP-tree mining

In this subsection, we examine some interesting properties of the FP-tree structure which will facilitate frequent-pattern mining.

Property 3.1 (*Node-link property*). *For any frequent item a_i, all the possible patterns containing only frequent items and a_i can be obtained by following a_i's node-links, starting from a_i's head in the FP-tree header.*

This property is directly from the FP-tree construction process, and it facilitates the access of all the frequent-pattern information related to a_i by traversing the FP-tree once following a_i's node-links.

To facilitate the understanding of other properties of FP-tree related to mining, we first go through an example which performs mining on the constructed FP-tree (figure 1) in Example 1.

Example 2. Let us examine the mining process based on the constructed FP-tree shown in figure 1. Based on Property 3.1, all the patterns containing frequent items that a node a_i participates can be collected by starting at a_i's node-link head and following its node-links. We examine the mining process by starting from the bottom of the node-link header table.

For node p, its immediate frequent pattern is $(p{:}3)$, and it has two paths in the FP-tree: $\langle f{:}4, c{:}3, a{:}3, m{:}2, p{:}2\rangle$ and $\langle c{:}1, b{:}1, p{:}1\rangle$. The first path indicates that string "(f, c, a, m, p)" appears twice in the database. Notice the path also indicates that string

$\langle f, c, a\rangle$ appears three times and $\langle f\rangle$ itself appears even four times. However, they only appear twice *together* with p. Thus, to study which string appear together with p, only p's prefix path $\langle f{:}2, c{:}2, a{:}2, m{:}2\rangle$ (or simply, $\langle fcam{:}2\rangle$) counts. Similarly, the second path indicates string "(c, b, p)" appears once in the set of transactions in *DB*, or p's prefix path is $\langle cb{:}1\rangle$. These two prefix paths of p, "$\{(fcam{:}2), (cb{:}1)\}$", form p's subpattern-base, which is called p's conditional pattern base (i.e., the subpattern-base under the condition of p's existence). Construction of an FP-tree on this conditional pattern-base (which is called p's conditional FP-tree) leads to only one branch $(c{:}3)$. Hence, only one frequent pattern $(cp{:}3)$ is derived. (Notice that a pattern is an itemset and is denoted by a string here.) The search for frequent patterns associated with p terminates.

For node m, its immediate frequent pattern is $(m{:}3)$, and it has two paths, $\langle f{:}4, c{:}3, a{:}3, m{:}2\rangle$ and $\langle f{:}4, c{:}3, a{:}3, b{:}1, m{:}1\rangle$. Notice p appears together with m as well, however, there is no need to include p here in the analysis since any frequent patterns involving p has been analyzed in the previous examination of p. Similar to the above analysis, m's conditional pattern-base is $\{(fca{:}2), (fcab{:}1)\}$. Constructing an FP-tree on it, we derive m's conditional FP-tree, $\langle f{:}3, c{:}3, a{:}3\rangle$, a single frequent pattern path, as shown in figure 3. This conditional FP-tree is then mined recursively by calling $mine(\langle f{:}3, c{:}3, a{:}3\rangle \mid m)$.

Figure 3 shows that "$mine(\langle f{:}3, c{:}3, a{:}3\rangle \mid m)$" involves mining three items (a), (c), (f) in sequence. The first derives a frequent pattern $(am{:}3)$, a conditional pattern-base $\{(fc{:}3)\}$, and then a call "$mine(\langle f{:}3, c{:}3\rangle \mid am)$"; the second derives a frequent pattern $(cm{:}3)$, a conditional pattern-base $\{(f{:}3)\}$, and then a call "$mine(\langle f{:}3\rangle \mid cm)$"; and the third derives only a frequent pattern $(fm{:}3)$. Further recursive call of "$mine(\langle f{:}3, c{:}3\rangle \mid am)$" derives two patterns $(cam{:}3)$ and $(fam{:}3)$, and a conditional pattern-base $\{(f{:}3)\}$, which then leads to a call "$mine(\langle f{:}3\rangle \mid cam)$", that derives the longest pattern $(fcam{:}3)$. Similarly, the call of "$mine(\langle f{:}3\rangle \mid cm)$" derives one pattern $(fcm{:}3)$. Therefore, the set of frequent patterns involving m is $\{(m{:}3), (am{:}3), (cm{:}3), (fm{:}3), (cam{:}3), (fam{:}3), (fcam{:}3), (fcm{:}3)\}$. This indicates that *a single path FP-tree can be mined by outputting all the combinations of the items in the path.*

Similarly, node b derives $(b{:}3)$ and it has three paths: $\langle f{:}4, c{:}3, a{:}3, b{:}1\rangle$, $\langle f{:}4, b{:}1\rangle$, and $\langle c{:}1, b{:}1\rangle$. Since b's conditional pattern-base $\{(fca{:}1), (f{:}1), (c{:}1)\}$ generates no frequent item, the mining for b terminates. Node a derives one frequent pattern $\{(a{:}3)\}$ and one subpattern base $\{(fc{:}3)\}$, a single-path conditional FP-tree. Thus, its set of frequent patterns

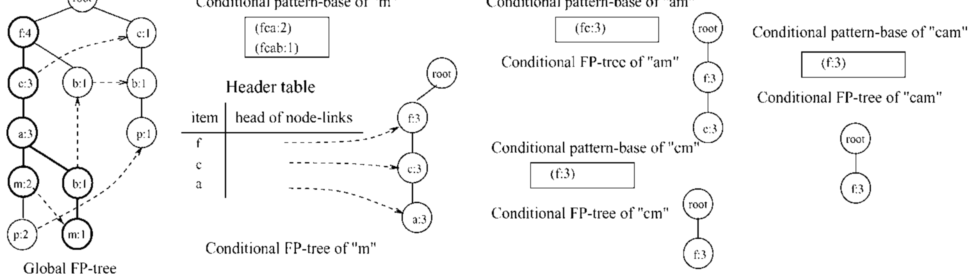

root
f:4
c:1
c:3
b:1
b:1
a:3
p:1
m:2
b:1
p:2
m:1
Global FP-tree
Conditional pattern-base of "m"
(fca:2)
(fcab:1)
Header table
item
head of node-links
f
c
a
root
f:3
c:3
a:3
Conditional FP-tree of "m"
Conditional pattern-base of "am"
(fc:3)
root
f:3
c:3
Conditional FP-tree of "am"
Conditional pattern-base of "cm"
(f:3)
Conditional FP-tree of "cm"
root
f:3
Conditional pattern-base of "cam"
(f:3)
Conditional FP-tree of "cam"
root
f:3

Figure 3. Mining *FP-tree* | m, a conditional FP-tree for item m.

Table 2. Mining frequent patterns by creating conditional (sub)pattern-bases.

Item	Conditional pattern-base	Conditional FP-tree
p	$\{(fcam{:}2), (cb{:}1)\}$	$\{(c{:}3)\}\|p$
m	$\{(fca{:}2), (fcab{:}1)\}$	$\{(f{:}3,\ c{:}3,\ a{:}3)\}\|m$
b	$\{(fca{:}1), (f{:}1), (c{:}1)\}$	$\emptyset$
a	$\{(fc{:}3)\}$	$\{(f{:}3,\ c{:}3)\}\|a$
c	$\{(f{:}3)\}$	$\{(f{:}3)\}\|c$
f	$\emptyset$	$\emptyset$

can be generated by taking their combinations. Concatenating them with (a:3), we have $\{(fa{:}3), (ca{:}3), (fca{:}3)\}$. Node c derives (c:4) and one subpattern-base $\{(f{:}3)\}$, and the set of frequent patterns associated with (c:3) is $\{(fc{:}3)\}$. Node f derives only (f:4) but no conditional pattern-base.

The conditional pattern-bases and the conditional FP-trees generated are summarized in Table 2.

The correctness and completeness of the process in Example 2 should be justified. This is accomplished by first introducing a few important properties related to the mining process.

Property 3.2 (*Prefix path property*). *To calculate the frequent patterns with suffix a_i, only the* prefix subpathes *of nodes labeled a_i in the FP-tree need to be accumulated*, *and the frequency count of every node in the prefix path should carry the same count as that in the corresponding node a_i in the path.*

Rationale. Let the nodes along the path P be labeled as $a_1, \ldots, a_n$ in such an order that a_1 is the root of the prefix subtree, a_n is the leaf of the subtree in P, and a_i $(1 \leq i \leq n)$ is the node being referenced. Based on the process of FP-tree construction presented in Algorithm 1, for each prefix node a_k $(1 \leq k < i)$, the prefix subpath of the node a_i in P occurs together with a_k exactly $a_i.count$ times. Thus every such prefix node should carry the same count as node a_i. Notice that a postfix node a_m (for $i < m \leq n$) along the same path also co-occurs with node a_i. However, the patterns with a_m will be generated when examining the suffix node a_m, enclosing them here will lead to redundant generation of the patterns that would have been generated for a_m. Therefore, we only need to examine the prefix subpath of a_i in P.

For example, in Example 2, node m is involved in a path $\langle f{:}4, c{:}3, a{:}3, m{:}2, p{:}2\rangle$, to calculate the frequent patterns for node m in this path, only the prefix subpath of node m, which is $\langle f{:}4, c{:}3, a{:}3\rangle$, need to be extracted, and the frequency count of every node in the prefix path should carry the same count as node m. That is, the node counts in the prefix path should be adjusted to $\langle f{:}2, c{:}2, a{:}2\rangle$.

Based on this property, the prefix subpath of node a_i in a path P can be copied and transformed into a count-adjusted prefix subpath by adjusting the frequency count of every node in the prefix subpath to the same as the count of node a_i. The prefix path so transformed is called the *transformed prefix path* of a_i for path P.

Notice that the set of transformed prefix paths of a_i forms a small database of patterns which co-occur with a_i. Such a database of patterns occurring with a_i is called *a_i's conditional pattern-base*, and is denoted as "*pattern_base* $\mid a_i$". Then one can compute all the frequent patterns associated with a_i in this a_i-conditional pattern-base by creating a small FP-tree, called *a_i's conditional FP-tree* and denoted as "FP-tree $\mid a_i$". Subsequent mining can be performed on this small conditional FP-tree. The processes of construction of conditional pattern-bases and conditional FP-trees have been demonstrated in Example 2.

This process is performed recursively, and the frequent patterns can be obtained by a pattern-growth method, based on the following lemmas and corollary.

Lemma 3.1 (*Fragment growth*). *Let α be an itemset in DB, B be α's conditional pattern-base, and β be an itemset in B. Then the support of $\alpha \cup \beta$ in DB is equivalent to the support of β in B.*

Rationale. According to the definition of conditional pattern-base, each (sub)transaction in B occurs under the condition of the occurrence of α in the original transaction database *DB*. If an itemset β appears in B ψ times, it appears with α in *DB* ψ times as well. Moreover, since all such items are collected in the conditional pattern-base of α, $\alpha \cup \beta$ occurs exactly ψ times in *DB* as well. Thus we have the lemma.

From this lemma, we can directly derive an important corollary.

Corollary 3.1 (*Pattern growth*). *Let α be a frequent itemset in DB, B be α's conditional pattern-base, and β be an itemset in B. Then $\alpha \cup \beta$ is frequent in DB if and only if β is frequent in B.*

Based on Corollary 3.1, mining can be performed by first identifying the set of frequent 1-itemsets in *DB*, and then for each such frequent 1-itemset, constructing its conditional pattern-bases, and mining its set of frequent 1-itemsets in the conditional pattern-base, and so on. This indicates that the process of mining frequent patterns can be viewed as first mining frequent 1-itemset and then progressively growing each such itemset by mining its conditional pattern-base, which can in turn be done similarly. By doing so, a frequent k-itemset mining problem is successfully transformed into a sequence of k frequent 1-itemset mining problems via a set of conditional pattern-bases. Since mining is done by pattern growth, there is no need to generate any candidate sets in the entire mining process.

Notice also in the construction of a new FP-tree from a *conditional pattern-base* obtained during the mining of an FP-tree, the items in the frequent itemset should be ordered in the frequency descending order of *node occurrence* of each item instead of its *support* (which represents item occurrence). This is because each node in an FP-tree may represent many occurrences of an item but such a node represents a single unit (i.e., the itemset whose elements always occur together) in the construction of an item-associated FP-tree.

3.2. Frequent-pattern growth with single prefix path of FP-tree

The frequent-pattern growth method described above works for all kinds of FP-trees. However, further optimization can be explored on a special kind of FP-tree, called *single prefix-path FP-tree*, and such an optimization is especially useful at mining long frequent patterns.

A single prefix-path FP-tree is an FP-tree that consists of only a single path or a single prefix path stretching from the root to the first branching node of the tree, where a *branching node* is a node containing more than one child.

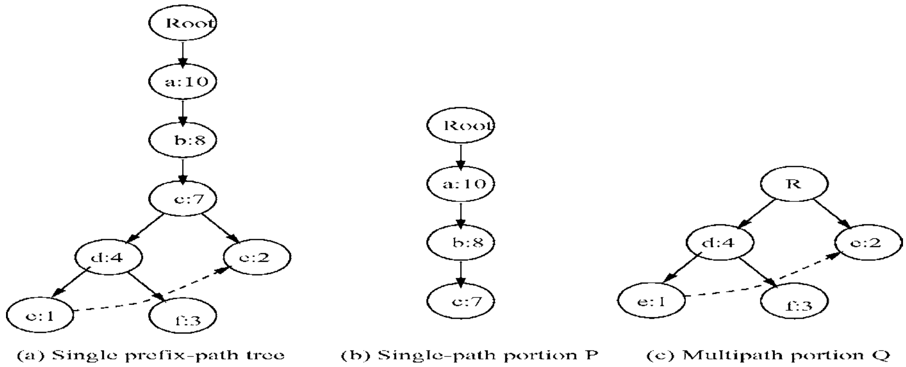
Let us examine an example.

Example 3. Figure 4(a) is a single prefix-path FP-tree that consists of one prefix path, $\langle (a{:}10) \rightarrow (b{:}8) \rightarrow (c{:}7) \rangle$, stretching from the root of the tree to the first branching node $(c{:}7)$. Although it can be mined using the frequent-pattern growth method described above, a better method is to split the tree into two fragments: the single prefix-path, $\langle (a{:}10) \rightarrow (b{:}8) \rightarrow (c{:}7) \rangle$, as shown in figure 4(b), and the multipath part, with the root replaced by a pseudo-root R, as shown in figure 4(c). These two parts can be mined separately and then combined together.

Let us examine the two separate mining processes. All the frequent patterns associated with the first part, the single prefix-path $P = \langle (a{:}10) \rightarrow (b{:}8) \rightarrow (c{:}7) \rangle$, can be mined by enumeration of all the combinations of the subpaths of P with the support set to the minimum support of the items contained in the subpath. This is because each such subpath is distinct and occurs the same number of times as the *minimum occurrence frequency among the items in the subpath* which is equal to the support of the last item in the subpath. Thus, path P generates the following set of frequent patterns, *freq_pattern_set*$(P) = \{(a{:}10), (b{:}8), (c{:}7), (ab{:}8), (ac{:}7), (bc{:}7), (abc{:}7)\}$.

Let Q be the second FP-tree (figure 4(c)), the multipath part rooted with R. Q can be mined as follows.

First, R is treated as a *null* root, and Q forms a multipath FP-tree, which can be mined using a typical frequent-pattern growth method. The mining result is: *freq_pattern_set*$(Q) = \{(d{:}4), (e{:}3), (f{:}3), (df{:}3)\}$.



(a) Single prefix-path tree (b) Single-path portion P (c) Multipath portion Q

Figure 4. Mining an FP-tree with a single prefix path.

Second, for each frequent itemset in Q, R can be viewed as a conditional frequent pattern-base, and each itemset in Q with each pattern generated from R may form a distinct frequent pattern. For example, for $(d{:}4)$ in *freq_pattern_set*(Q), P can be viewed as its conditional pattern-base, and a pattern generated from P, such as $(a{:}10)$, will generate with it a new frequent itemset, $(ad{:}4)$, since a appears together with d at most four times. Thus, for $(d{:}4)$ the set of frequent patterns generated will be $(d{:}4) \times$ *freq_pattern_set*$(P) =$ $\{(ad{:}4), (bd{:}4), (cd{:}4), (abd{:}4), (acd{:}4), (bcd{:}4), (abcd{:}4)\}$, where $X \times Y$ means that every pattern in X is combined with every one in Y to form a "cross-product-like" larger itemset with the support being the minimum support between the two patterns. Thus, the complete set of frequent patterns generated by combining the results of P and Q will be *freq_pattern_set*$(Q) \times$ *freq_pattern_set*(P), with the support being the support of the itemset in Q (which is always no more than the support of the itemset from P).

In summary, the set of frequent patterns generated from such a single prefix path consists of three distinct sets: (1) *freq_pattern_set*(P), the set of frequent patterns generated from the single prefix-path, P; (2) *freq_pattern_set*(Q), the set of frequent patterns generated from the multipath part of the FP-tree, Q; and (3) *freq_pattern_set*$(Q) \times$ *freq_pattern_set*(P), the set of frequent patterns involving both parts.

We first show if an FP-tree consists of a single path P, one can generate the set of frequent patterns according to the following lemma.

Lemma 3.2 (*Pattern generation for an FP-tree consisting of single path*)**.** *Suppose an FP-tree T consists of a single path P. The complete set of the frequent patterns of T can be generated by enumeration of all the combinations of the subpaths of P with the support being the minimum support of the items contained in the subpath.*

Rationale. Let the single path P of the FP-tree be $\langle a_1{:}s_1 \rightarrow a_2{:}s_2 \rightarrow \cdots \rightarrow a_k{:}s_k\rangle$. Since the FP-tree contains a single path P, the support frequency s_i of each item a_i (for $1 \leq i \leq k$) is the frequency of a_i co-occurring with its prefix string. Thus, any combination of the items in the path, such as $\langle a_i, \ldots, a_j\rangle$ (for $1 \leq i, j \leq k$), is a frequent pattern, with their co-occurrence frequency being the minimum support among those items. Since every item in each path P is unique, there is no redundant pattern to be generated with such a combinational generation. Moreover, no frequent patterns can be generated outside the FP-tree. Therefore, we have the lemma.

We then show if an FP-tree consists of a single prefix-path, the set of frequent patterns can be generated by splitting the tree into two according to the following lemma.

Lemma 3.3 (*Pattern generation for an FP-tree consisting of single prefix path*)**.** *Suppose an FP-tree T, similar to the tree in figure* 4(a), *consists of* (1) *a single prefix path P, similar to the tree P in figure* 4(b), *and* (2) *the multipath part, Q, which can be viewed as an independent FP-tree with a pseudo-root R, similar to the tree Q in figure* 4(c).

The complete set of the frequent patterns of T consists of the following three portions:

1. *The set of frequent patterns generated from P by enumeration of all the combinations of the items along path P, with the support being the minimum support among all the items that the pattern contains.*
2. *The set of frequent patterns generated from Q by taking root R as "null."*
3. *The set of frequent patterns combining P and Q formed by taken the cross-product of the frequent patterns generated from P and Q, denoted as freq_pattern_set(P) × freq_pattern_set(Q), that is, each frequent itemset is the union of one frequent itemset from P and one from Q and its support is the minimum one between the supports of the two itemsets.*

Rationale. Based on the FP-tree construction rules, each node a_i in the single prefix path of the FP-tree appears only once in the tree. The single prefix-path of the FP-tree forms a new FP-tree P, and the multipath part forms another FP-tree Q. They do not share nodes representing the same item. Thus, the two FP-trees can be mined separately.

First, we show that each pattern generated from one of the three portions by following the pattern generation rules is distinct and frequent. According to Lemma 3.2, each pattern generated from P, the FP-tree formed by the single prefix-path, is distinct and frequent. The set of frequent patterns generated from Q by taking root R as "null" is also distinct and frequent since such patterns exist without combining any items in their conditional databases (which are in the items in P. The set of frequent patterns generated by combining P and Q, that is, taking the cross-product of the frequent patterns generated from P and Q, with the support being the minimum one between the supports of the two itemsets, is also distinct and frequent. This is because each frequent pattern generated by P can be considered as a frequent pattern in the conditional pattern-base of a frequent item in Q, and whose support should be the minimum one between the two supports since this is the frequency that both patterns appear together.

Second, we show that no patterns can be generated out of this three portions. Since according to Lemma 3.1, the FP-tree T without being split into two FP-trees P and Q generates the complete set of frequent patterns by pattern growth. Since each pattern generated from T will be generated from either the portion P or Q or their combination, the method generates the complete set of frequent patterns.

3.3. The frequent-pattern growth algorithm

Based on the above lemmas and properties, we have the following algorithm for mining frequent patterns using FP-tree.

Algorithm 2 (FP-growth*: Mining frequent patterns with FP-tree by pattern fragment growth*)**.**

Input: A database *DB*, represented by FP-tree constructed according to Algorithm 1, and a minimum support threshold ξ.
Output: The complete set of frequent patterns.

Method: *call FP-growth*(FP-tree, *null*).

```
Procedure FP-growth(Tree, $\alpha$)
{
(1)  if Tree contains a single prefix path    // Mining single prefix-path FP-tree
(2)  then {
(3)      let $P$ be the single prefix-path part of Tree;
(4)      let $Q$ be the multipath part with the top branching node replaced by a null root;
(5)      for each combination (denoted as $\beta$) of the nodes in the path $P$ do
(6)          generate pattern $\beta \cup \alpha$ with $support = minimum\ support\ of\ nodes\ in\ \beta$;
(7)      let freq_pattern_set($P$) be the set of patterns so generated;     }
(8)  else let $Q$ be Tree;
(9)  for each item $a_i$ in $Q$ do {                        // Mining multipath FP-tree
(10)     generate pattern $\beta = a_i \cup \alpha$ with $support = a_i.support$;
(11)     construct $\beta$'s conditional pattern-base and then $\beta$'s conditional FP-tree $Tree_\beta$;
(12)     if $Tree_\beta \neq \emptyset$
(13)     then call FP-growth($Tree_\beta$, $\beta$);
(14)     let freq_pattern_set($Q$) be the set of patterns so generated;     }
(15) return(freq_pattern_set($P$) $\cup$ freq_pattern_set($Q$) $\cup$ (freq_pattern_set($P$)
         $\times$ freq_pattern_set($Q$)))
}
```

Analysis. With the properties and lemmas in Sections 2 and 3, we show that the algorithm correctly finds the complete set of frequent itemsets in transaction database *DB*.

As shown in Lemma 2.1, FP-tree of *DB* contains the complete information of *DB* in relevance to frequent pattern mining under the support threshold ξ.

If an FP-tree contains a single prefix-path, according to Lemma 3.3, the generation of the complete set of frequent patterns can be partitioned into three portions: the single prefix-path portion P, the multipath portion Q, and their combinations. Hence we have lines (1)-(4) and line (15) of the procedure. According to Lemma 3.2, the generated patterns for the single prefix path are the enumerations of the subpaths of the prefix path, with the support being the minimum support of the nodes in the subpath. Thus we have lines (5)-(7) of the procedure. After that, one can treat the multipath portion or the FP-tree that does not contain the single prefix-path as portion Q (lines (4) and (8)) and construct conditional pattern-base and mine its conditional FP-tree for each frequent itemset a_i. The correctness and completeness of the prefix path transformation are shown in Property 3.2. Thus the conditional pattern-bases store the complete information for frequent pattern mining for Q. According to Lemmas 3.1 and its corollary, the patterns successively grown from the conditional FP-trees are the set of sound and complete frequent patterns. Especially, according to the fragment growth property, the support of the combined fragments takes the support of the frequent itemsets generated in the conditional pattern-base. Therefore, we have lines (9)-(14) of the procedure. Line (15) sums up the complete result according to Lemma 3.3.

Let's now examine the efficiency of the algorithm. The *FP-growth* mining process scans the FP-tree of *DB* once and generates a small pattern-base B_{a_i} for each frequent item a_i, each consisting of the set of transformed prefix paths of a_i. Frequent pattern mining is then recursively performed on the small pattern-base B_{a_i} by constructing a conditional FP-tree for B_{a_i}. As reasoned in the analysis of Algorithm 1, an FP-tree is usually much smaller than the size of *DB*. Similarly, since the conditional FP-tree, "FP-tree | a_i", is constructed on the pattern-base B_{a_i}, it should be usually much smaller and never bigger than B_{a_i}. Moreover, a pattern-base B_{a_i} is usually much smaller than its original FP-tree, because it consists of the transformed prefix paths related to only one of the frequent items, a_i. Thus, each subsequent mining process works on a set of usually much smaller pattern-bases and conditional FP-trees. Moreover, the mining operations consist of mainly prefix count adjustment, counting local frequent items, and pattern fragment concatenation. This is much less costly than generation and test of a very large number of candidate patterns. Thus the algorithm is efficient.

From the algorithm and its reasoning, one can see that the *FP-growth* mining process is a divide-and-conquer process, and the scale of shrinking is usually quite dramatic. If the shrinking factor is around 20-100 for constructing an FP-tree from a database, it is expected to be another hundreds of times reduction for constructing each conditional FP-tree from its already quite small conditional frequent pattern-base.

Notice that even in the case that a database may generate an exponential number of frequent patterns, the size of the FP-tree is usually quite small and will never grow exponentially. For example, for a frequent pattern of length 100, "$a_1, \ldots, a_{100}$", the FP-tree construction results in only one path of length 100 for it, possibly "$\langle a_1, \rightarrow \cdots \rightarrow a_{100} \rangle$" (if the items are ordered in the *list* of frequent items as $a_1, \ldots, a_{100}$). The *FP-growth* algorithm will still generate about 10^{30} frequent patterns (if time permits!!), such as "$a_1, a_2, \ldots, a_1a_2, \ldots, a_1a_2a_3, \ldots, a_1 \ldots a_{100}$." However, the FP-tree contains only one frequent pattern path of 100 nodes, and according to Lemma 3.2, there is even no need to construct any conditional FP-tree in order to find all the patterns.

4. Scaling FP-tree-based *FP-growth* by database projection

FP-growth proposed in the last section is essentially a main memory-based frequent pattern mining method. However, when the database is large, or when the minimum support threshold is quite low, it is unrealistic to assume that the FP-tree of a database can fit in main memory. A disk-based method should be worked out to ensure that mining is highly scalable. In this section, a method is developed to first partition the database into a set of projected databases, and then for each projected database, construct and mine its corresponding FP-tree.

Let us revisit the mining problem in Example 1.

Example 4. Suppose the FP-tree in figure 1 cannot be held in main memory. Instead of constructing a global FP-tree, one can project the transaction database into a set of frequent item-related *projected databases* as follows.

Starting at the tail of the frequent item list, p, the set of transactions that contain item p can be collected into *p-projected database*. Infrequent items and item p itself can be removed from them because the infrequent items are not useful in frequent pattern mining,

Table 3. Projected databases and their FP-trees.

Item	Projected database	Conditional FP-tree
p	{*fcam*, *cb*, *fcam*}	$\{(c{:}3)\}\|p$
m	{*fca*, *fcab*, *fca*}	$\{(f{:}3,\ c{:}3,\ a{:}3)\}\|m$
b	{*fca*, f, c}	$\emptyset$
a	{*fc*, *fc*, *fc*}	$\{(f{:}3,\ c{:}3)\}\|a$
c	$\{f,\ f,\ f\}$	$\{(f{:}3)\}\|c$
f	$\emptyset$	$\emptyset$

and item p is by default associated with each projected transaction. Thus, the p-projected database becomes {*fcam*, *cb*, *fcam*}. This is very similar to the the p-conditional pattern-base shown in Table 2 except *fcam* and *fcam* are expressed as (*fcam*:2) in Table 2. After that, the p-conditional FP-tree can be built on the p-projected database based on the FP-tree construction algorithm.

Similarly, the set of transactions containing item m can be projected into *m-projected database*. Notice that besides infrequent items and item m, item p is also excluded from the set of projected items because item p and its association with m have been considered in the p-projected database. For the same reason, the *b-projected database* is formed by collecting transactions containing item b, but infrequent items and items f, m and b are excluded. This process continues for deriving *a-projected database*, *c-projected database*, and so on. The complete set of item-projected databases derived from the transaction database are listed in Table 3, together with their corresponding conditional FP-trees. One can easily see that the two processes, *construction of the global FP-tree* and *projection of the database into a set of projected databases*, derive identical conditional FP-trees.

As shown in Section 2, a conditional FP-tree is usually orders of magnitude smaller than the global FP-tree. Thus, construction of a conditional FP-tree from each projected database and then mining on it will dramatically reduce the size of FP-trees to be handled. What about that a conditional FP-tree of a projected database still cannot fit in main memory? One can further project the projected database, and the process can go on recursively until the conditional FP-tree fits in main memory.

Let us define the concept of projected database formally.

Definition 2 (Projected database).

- Let a_i be a frequent item in a transaction database, *DB*. The *a_i-projected database* for a_i is derived from *DB* by collecting all the transactions containing a_i and removing from them (1) infrequent items, (2) all frequent items after a_i in the *list of frequent items*, and (3) a_i itself.
- Let a_j be a frequent item in α-projected database. Then the *$a_j\alpha$-projected database* is derived from the α-projected database by collecting all entries containing a_j and removing

from them (1) infrequent items, (2) all frequent items after a_j in the *list of frequent items*, and (3) a_j itself.

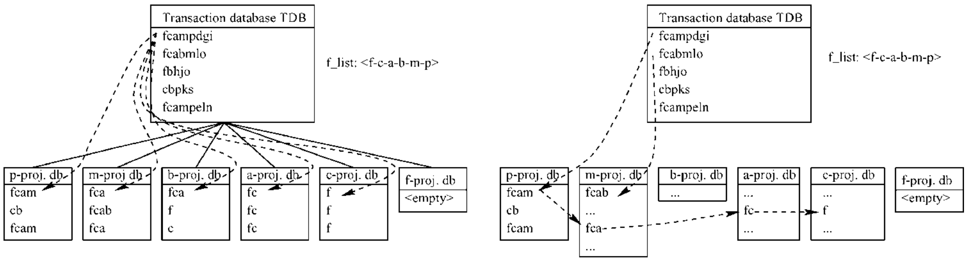
According to the rules of construction of FP-tree and that of construction of projected database, the a_i-projected database is derived by projecting the same set of items in the transactions containing a_i into the projected database as those collected in the construction of the a_i-subtree in the FP-tree. Thus, the two methods derive the same sets of conditional FP-trees.

There are two methods for database projection: *parallel projection* and *partition projection*.

Parallel projection is implemented as follows: Scan the database to be projected once, where the database could be either a transaction database or an α-projected database. For each transaction T in the database, for each frequent item a_i in T, project T to the a_i-projected database based on the transaction projection rule, specified in the definition of projected database. Since a transaction is projected in parallel to all the projected databases in one scan, it is called *parallel projection*. The set of projected databases shown in Table 3 of Example 4 demonstrates the result of parallel projection. This process is illustrated in figure 5(a).

Parallel projection facilitates parallel processing because all the projected databases are available for mining at the end of the scan, and these projected databases can be mined in parallel. However, since each transaction in the database is projected to multiple projected databases, if a database contains many long transactions with multiple frequent items, the total size of the projected databases could be multiple times of the original one. Let each transaction contains on average l frequent items. A transaction is then projected to $l - 1$ projected database. The total size of the projected data from this transaction is $1 + 2 + \cdots + (l - 1) = \frac{l(l-1)}{2}$. This implies that the total size of the single item-projected databases is about $\frac{l-1}{2}$ times of that of the original database.

To avoid such an overhead, we propose a *partition projection* method. Partition projection is implemented as follows. When scanning the database (original or α-projected) to be projected, a transaction T is *projected* to the a_i-projected database only if a_i is a frequent item in T and there is no any other item after a_i in the *list of frequent items* appearing



(a) Parallel projection

(b) Partition Projection

Figure 5. Parallel projection vs. partition projection.

in the transaction. Since a transaction is projected to only one projected database at the database scan, after the scan, the database is partitioned by projection into a set of projected databases, and hence it is called *partition projection*.

The projected databases are mined in the reversed order of the *list of frequent items*. That is, the projected database of the least frequent item is mined first, and so on. Each time when a projected database is being processed, to ensure the remaining projected databases obtain the complete information, each transaction in it is projected to the a_j-projected database, where a_j is the item in the transaction such that there is no any other item after a_j in the *list of frequent items* appearing in the transaction. The partition projection process for the database in Example 4 is illustrated in figure 5(b).

The advantage of partition projection is that the total size of the projected databases at each level is smaller than the original database, and it usually takes less memory and I/Os to complete the partition projection. However, the processing order of the projected databases becomes important, and one has to process these projected databases in a sequential manner. Also, during the processing of each projected database, one needs to project the processed transactions to their corresponding projected databases, which may take some I/O as well. Nevertheless, due to its low memory requirement, partition projection is still a promising method in frequent pattern mining.

Example 5. Let us examine how the database in Example 4 can be projected by partition projection.

First, by one scan of the transaction database, each transaction is projected to only one projected database. The first transaction, *facdgimp*, is projected to the *p-projected database* since *p* is the last frequent item in the *list of frequent items*. Thus, *fcam* (i.e., with infrequent items removed) is inserted into the *p*-projected database. Similarly, transaction *abcflmo* is projected to the *m-projected database* as *fcab*, *bfhjo* to the *b-projected database* as *f*, *bcksp* to the *p*-projected database as *cb*, and finally, *afcelpmn* to the *p*-projected database as *fcam*. After this phrase, the entries in every projected databases are shown in Table 4.

With this projection, the original database can be replaced by the set of single-item projected databases, and the total size of them is smaller than that of the original database.

Second, the *p*-projected database is first processed (i.e., construction of *p*-conditional FP-tree), where *p* is the last item in the *list of frequent items*. During the processing of the *p*-projected database, each transaction is projected to the corresponding projected database

Table 4. Single-item projected databases by partition projection.

Item	Projected databases
p	$\{fcam, cb, fcam\}$
m	$\{fcab\}$
b	$\{f\}$
a	$\emptyset$
c	$\emptyset$
f	$\emptyset$

according to the same partition projection rule. For example, *fcam* is projected to the *m*-projected database as *fca*, *cb* is projected to the *b*-projected database as *c*, and so on. The process continues until every single-item projected database is completely processed.

5. Experimental evaluation and performance study

In this section, we present a performance comparison of *FP-growth* with the classical frequent pattern mining algorithm *Apriori*, and an alternative database projection-based algorithm, *TreeProjection*. We first give a concise introduction and analysis to *TreeProjection*, and then report our experimental results.

5.1. A comparative analysis of FP-growth and TreeProjection methods

The *TreeProjection* algorithm proposed by Agarwal et al. (2001) constructs a lexicographical tree and projects a large database into a set of reduced, item-based sub-databases based on the frequent patterns mined so far. Since it applies a tree construction method and performs mining recursively on progressively smaller databases, it shares some similarities with *FP-growth*. However, the two methods have some fundamental differences in tree construction and mining methodologies, and will lead to notable differences on efficiency and scalability. We will explain such similarities and differences by working through the following example.

Example 6. For the same transaction database presented in Example 1, we construct the lexicographic tree according to the method described in Agarwal et al. (2001). The result tree is shown in figure 6, and the construction process is presented as follows.

By scanning the transaction database once, all frequent 1-itemsets are identified. As recommended in Agarwal et al. (2001), the frequency ascending order is chosen as the

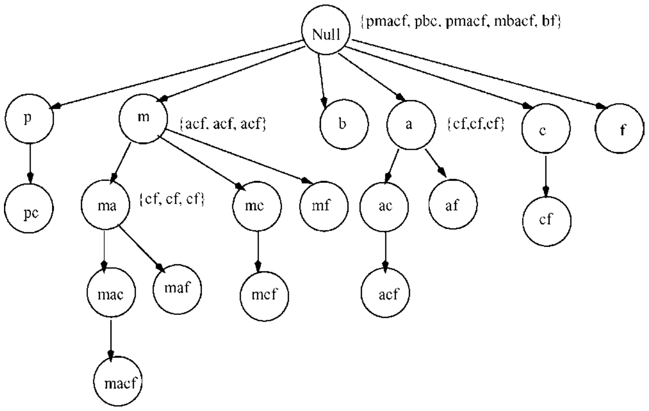


Figure 6. A lexicographical tree built for the same transactional database *DB*.

ordering of the items. So, the order is *p*-*m*-*b*-*a*-*c*-*f*, which is exactly the reverse order of what is used in the FP-tree construction. The top level of the lexicographic tree is constructed, i.e. the root and the nodes labeled by length-1 patterns. At this stage, the root node labeled "null" and all the nodes which store frequent 1-itemsets are generated. All the transactions in the database are projected to the root node, i.e., all the infrequent items are removed.

Each node in the lexicographical tree contains two pieces of information: (i) the pattern that node represents, and (ii) the set of items that may generate longer patterns by adding them to the pattern. The latter piece information is recorded as *active extensions* and *active items*.

Then, a matrix at the root node is created, as shown below. The matrix computes the frequencies of length-2 patterns, thus all pairs of frequent items are included in the matrix. The items in pairs are arranged in the ordering. The matrix is built by adding counts from every transaction, i.e., computing frequent 2-itemsets based on transactions stored in the root node.

$$\begin{array}{c|cccccc}
 & p & m & b & a & c & f \\
p & & & & & & \\
m & 2 & & & & & \\
b & 1 & 1 & & & & \\
a & 2 & 3 & 1 & & & \\
c & 3 & 3 & 2 & 3 & & \\
f & 2 & 3 & 2 & 3 & 3 &
\end{array}$$

At the same time of building the matrix, transactions in the root are projected to level-1 nodes as follows. Let $t = a_1a_2\ldots a_n$ be a transaction with all items listed in ordering. t is projected to node a_i $(1 \leq i < n - 1)$ as $t'_{a_i} = a_{i+1}a_{i+2}\ldots a_n$.

From the matrix, all the frequent 2-itemsets are found as: $\{pc, ma, mc, mf, ac, af, cf\}$. The nodes in lexicographic tree for them are generated. At this stage, the only nodes for 1-itemsets which are active are those for m and a, because only they contain enough descendants to potentially generate longer frequent itemsets. All nodes up to and including level-1 except for these two nodes are pruned.

In the same way, the lexicographic tree is grown level by level. From the matrix at node m, nodes labeled mac, maf, and mcf are added, and only ma is active in all the nodes for frequent 2-itemsets. It is easy to see that the lexicographic tree in total contains 19 nodes.

The number of nodes in a lexicographic tree is exactly that of the frequent itemsets. *TreeProjection* proposes an efficient way to enumerate frequent patterns. The efficiency of *TreeProjection* can be explained by two main factors: (1) the transaction projection limits the support counting in a relatively small space, and only related portions of transactions are considered; and (2) the lexicographical tree facilitates the management and counting of candidates and provides the flexibility of picking efficient strategy during the tree generation phase as well as transaction projection phase. Agarwal et al. (2001) reports that their algorithm is up to one order of magnitude faster than other recent techniques in literature.

However, in comparison with the *FP-growth* method, *TreeProjection* suffers from some problems related to efficiency, and scalability. We analyze them as follows.

First, *TreeProjection* may encounter difficulties at computing matrices when the database is huge, when there are a lot of transactions containing many frequent items, and/or when the support threshold is very low. This is because in such cases there often exist a large number of frequent items. The size of the matrices at high level nodes in the lexicographical tree can be huge, as shown in our introduction section. The study in *TreeProjection* (Agarwal et al., 2001) has developed some smart memory caching methods to overcome this problem. However, it could be wise not to generate such huge matrices at all instead of finding some smart caching techniques to reduce the cost. Moreover, even if the matrix can be cached efficiently, its computation still involves some nontrivial overhead. To compute a matrix at node P with n projected transactions, the cost is $O(\sum_{i=1}^{n} \frac{|T_i|^2}{2})$, where $|T_i|$ is the length of the transaction. If the number of transaction is large and the length of each transaction is long, the computation is costly. The *FP-growth* method will never need to build up matrices and compute 2-itemset frequency since it avoids the generation of any candidate k-itemsets for any k by applying a pattern growth method. Pattern growth can be viewed as successive computation of frequent 1-itemset (of the database and conditional pattern bases) and assembling them into longer patterns. Since computing frequent 1-itemsets is much less expensive than computing frequent 2-itemsets, the cost is substantially reduced.

Second, since one transaction may contain many frequent itemsets, one transaction in *TreeProjection* may be projected many times to many different nodes in the lexicographical tree. When there are many long transactions containing numerous frequent items, transaction projection becomes a nontrivial cost of *TreeProjection*. The *FP-growth* method constructs FP-tree which is a highly compact form of transaction database. Thus both the size and the cost of computation of conditional pattern bases, which corresponds roughly to the compact form of projected transaction databases, are substantially reduced.

Third, *TreeProjection* creates one node in its lexicographical tree for each frequent itemset. At the first glance, this seems to be highly compact since FP-tree does not ensure that each frequent node will be mapped to only one node in the tree. However, each branch of the FP-tree may store many "hidden" frequent patterns due to the potential generation of many combinations using its prefix paths. Notice that the total number of frequent k-itemsets can be very large in a large database or when the database has quite long frequent itemsets. For example, for a frequent itemset $(a_1, a_2, \ldots, a_{100})$, the number of frequent itemsets at the 50th-level of the lexicographic tree will be $\binom{100}{50} = \frac{100!}{50! \times 50!} \approx 1.0 \times 10^{29}$. For the same frequent itemset, FP-tree and *FP-growth* will only need one path of 100 nodes.

In summary, *FP-growth* mines frequent itemsets by (1) constructing highly compact FP-trees which share numerous "projected" transactions and hide (or carry) numerous frequent patterns, and (2) applying progressive pattern growth of frequent 1-itemsets which avoids the generation of any potential combinations of candidate itemsets implicitly or explicitly, whereas *TreeProjection* must generate candidate 2-itemsets for each projected database. Therefore, *FP-growth* is more efficient and more scalable than *TreeProjection*, especially when the number of frequent itemsets becomes really large. These observations and analyses are well supported by our experiments reported in this section.

5.2. *Environments of experiments*

All the experiments are performed on a 266-MHz Pentium PC machine with 128 megabytes main memory, running on Microsoft Windows/NT. All the programs are written in Microsoft/Visual C++6.0. Notice that we do not directly compare our absolute number of runtime with those in some published reports running on the RISC workstations because different machine architectures may differ greatly on the absolute runtime for the same algorithms. Instead, we implement their algorithms to the best of our knowledge based on the published reports on the same machine and compare in the same running environment. Please also note that *run time* used here means the total execution time, that is, the period between input and output, instead of *CPU time* measured in the experiments in some literature. We feel that run time is a more comprehensive measure since it takes the total running time consumed as the measure of cost, whereas CPU time considers only the cost of the CPU resource. Also, all reports on the runtime of *FP-growth* include the time of constructing FP-trees from the original databases.

The experiments are pursued on both synthetic and real data sets. The synthetic data sets which we used for our experiments were generated using the procedure described in Agrawal and Srikant (1994). We refer readers to it for more details on the generation of data sets.

We report experimental results on two synthetic data sets. The first one is T10.I4.D100K with 1K items. In this data set, the average transaction size and average maximal potentially frequent itemset size are set to 10 and 4, respectively, while the number of transactions in the dataset is set to 100 K. It is a sparse dataset. The frequent itemsets are short and not numerous.

The second synthetic data set we used is T25.I20.D100K with 10 K items. The average transaction size and average maximal potentially frequent itemset size are set to 25 and 20, respectively. There exist exponentially numerous frequent itemsets in this data set when the support threshold goes down. There are also pretty long frequent itemsets as well as a large number of short frequent itemsets in it. It contains abundant mixtures of short and long frequent itemsets.

To test the capability of *FP-growth* on dense datasets with long patterns, we use the real data set *Connect-4*, compiled from the *Connect-4* game state information. The data set is from the UC-Irvine Machine Learning Database Repository (http://www.ics.uci.edu/∼mlearn/MLRepository.html). It contains 67, 557 transactions, while each transaction is with 43 items. It is a dense dataset with a lot of long frequent itemsets.

5.3. *Compactness of FP-tree*

To test the compactness of FP-trees, we compare the sizes of the following structures.

- Alphabetical FP-tree. It includes the space of all the links. However, in such an FP-tree, the alphabetical order of items are used instead of frequency descending order.
- Ordered FP-tree. Again, the size covers that of all links. In such an FP-tree, the items are sorted according to frequency descending order.

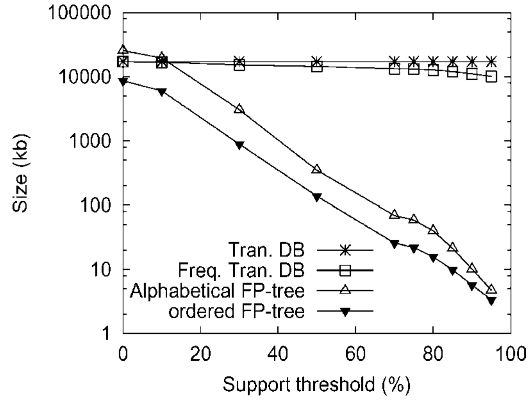


Figure 7. Compactness of FP-tree over data set *Connect-4*.

- Transaction database. Each item in a transaction is stored as an integer. It is simply the sum of occurrences of items in transactions.
- Frequent transaction database. That is the sub-database extracted from the original one by removing all infrequent items.

In real dataset *Connect-4*, FP-tree achieves good compactness. As seen from the result shown in figure 7, the size of ordered FP-tree is always smaller than the size of the transaction database and the frequent transaction database. In a dense database, the size of the database and that of its frequent database are close. The size of the alphabetical FP-tree is smaller than that of the two databases in most cases but is slightly larger (about 1.5 to 2.5 times larger) than the size of the ordered FP-tree. It indicates that the frequency-descending ordering of the items benefits data compression in this case.

In dataset T25.I20.D100k, which contains abundant mixture of long and short frequent patterns, FP-tree is compact most of the time. The result is shown in figure 8. Only when

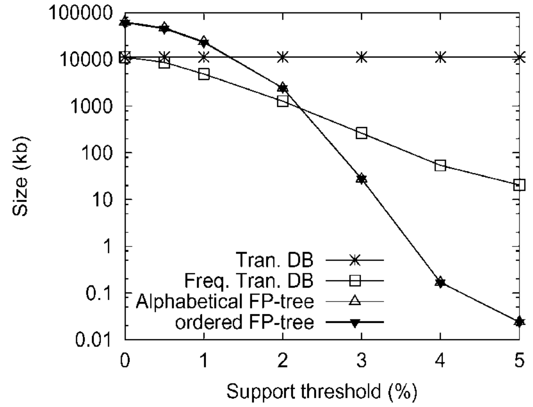


Figure 8. Compactness of FP-tree over data set T25.I20.D100k.

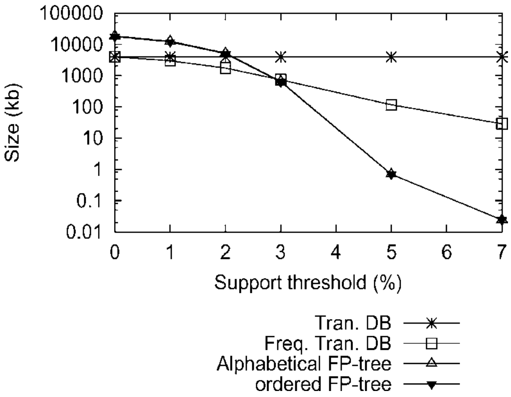


Figure 9. Compactness of FP-tree over data set T10.I4.D100k.

the support threshold lower than 2.5%, is the size of FP-tree larger than that of frequent database. Moreover, as long as the support threshold is over 1.5%, the FP-tree is smaller than the transaction database. The difference of sizes of ordered FP-tree and alphabetical FP-tree is quite small in this dataset. It is about 2%.

In sparse dataset T10.I4.D100k, FP-tree achieves good compactness when the support threshold is over 3.5%. Again, the difference of ordered FP-tree and alphabetical FP-tree is trivial. The result is shown in figure 9.

The above experiments lead to the following conclusions.

- FP-tree achieves good compactness most of the time. Especially in dense datasets, it can compress the database many times. Clearly, there is some overhead for pointers and counters. However, the gain of sharing among frequent projections of transactions is substantially more than the overhead and thus makes FP-tree space more efficient in many cases.
- When support is very low, FP-tree becomes bushy. In such cases, the degree of sharing in branches of FP-tree becomes low. The overhead of links makes the size of FP-tree large. Therefore, instead of building FP-tree, we should construct projected databases. That is the reason why we build FP-tree for transaction database/projected database only when it passes certain density threshold. From the experiments, one can see that such a threshold is pretty low, and easy to touch. Therefore, even for very large and/or sparse database, after one or a few rounds of database projection, FP-tree can be used for all the remaining mining tasks.

In the following experiments, we employed an implementation of *FP-growth* that integrates both database projection and FP-tree mining. The density threshold is set to 3%, and items are listed in frequency descending order.

5.4. Scalability study

The runtime of *Apriori*, *TreeProjection*, and *FP-growth* on synthetic data set T10.I4.D100K as the support threshold decreases from 0.15% to 0.01% is shown in figure 10.

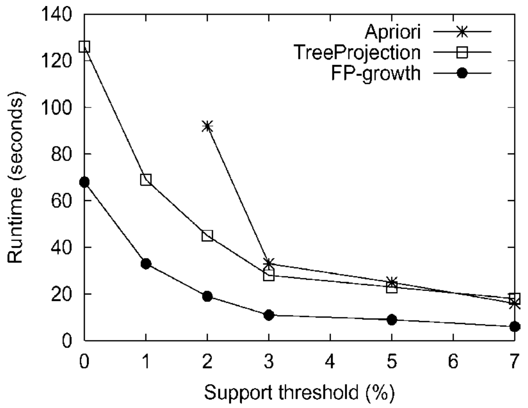


Figure 10. Scalability with threshold over sparse data set.

FP-growth is faster than both *Apriori* and *TreeProjection*. *TreeProjection* is faster and more scalable than *Apriori*. Since the dataset is sparse, as the support threshold is high, the frequent itemsets are short and the set of such itemsets is not large, the advantages of *FP-growth* and *TreeProjection* over *Apriori* are not so impressive. However, as the support threshold goes down, the gap becomes wider. *FP-growth* can finish the computation for support threshold 0.01% within the time for *Apriori* over 0.05%. *TreeProjection* is also scalable, but is slower than *FP-growth*.

The advantages of *FP-growth* over *Apriori* becomes obvious when the dataset contains an abundant number of mixtures of short and long frequent patterns. Figure 11 shows the experimental results of scalability with threshold over dataset T25.I20.D100k. *FP-growth* can mine with support threshold as low as 0.05%, with which *Apriori* cannot work out within reasonable time. *TreeProjection* is also scalable and faster than *Apriori*, but is slower than *FP-growth*.

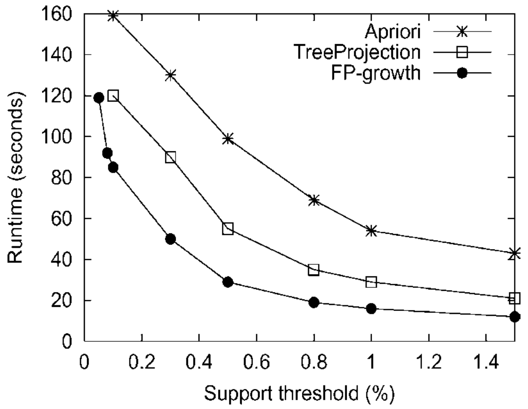


Figure 11. Scalability with threshold over dataset with abundant mixtures of short and long frequent patterns.

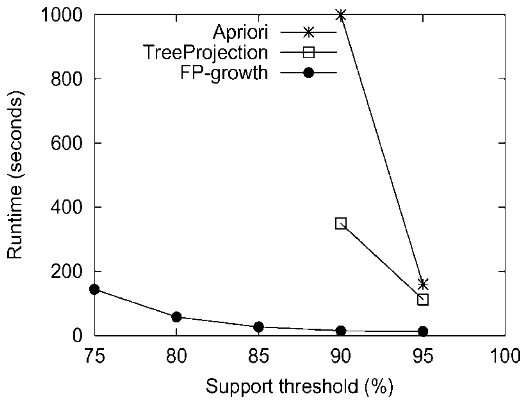


Figure 12. Scalability with threshold over *Connect-4*.

The advantage of *FP-growth* is dramatic in datasets with long patterns, which is challenging to the algorithms that mine the complete set of frequent patterns. The result on mining the real dataset *Connect-4* is shown in figure 12. To the best of our knowledge, this is the first algorithm that handles such dense real dataset in performance study. From the figure, one can see that *FP-growth* is scalable even when there are many long patterns. Without candidate generation, *FP-growth* enumerates long patterns efficiently. In such datasets, neither *Apriori* nor *TreeProjection* are comparable to the performance of *FP-growth*. To deal with long patterns, *Apriori* has to generate a tremendous number of candidates, that is very costly. The main costs in *TreeProjection* are matrix computation and transaction projection. In a database with a large number of frequent items, the matrices become quite large, and the computation cost jumps up substantially. In contrast, the height of FP-tree is limited by the maximal length of the transactions, and many transactions share the prefix paths of an FP-tree. This explains why *FP-growth* has distinct advantages when the support threshold is low and when the number of transactions is large.

To test the scalability of *FP-growth* against the number of transactions, a set of synthetic datasets are generated using the same parameters of T10.I4 and T25.I20, and the number of transactions ranges from 100 k to 1 M. *FP-growth* is tested over them using the same support threshold in percentage. The result is in figure 13, which shows the linear increase of runtime with the number of transactions. Please note that unlike the way reported in some literature, we do not replicate transactions in real data sets to test the scalability. This is because no matter how many times the transactions are replicated, *FP-growth* builds up an FP-tree with the size identical to that of the original (nonreplicated) one, and the scaling-up of such databases becomes trivial.

6. Discussions

The frequent-pattern growth method introduced here represents an interesting approach for scalable frequent-pattern mining. In this section, we discuss some additional issues related to its implementation, usage, and extension.

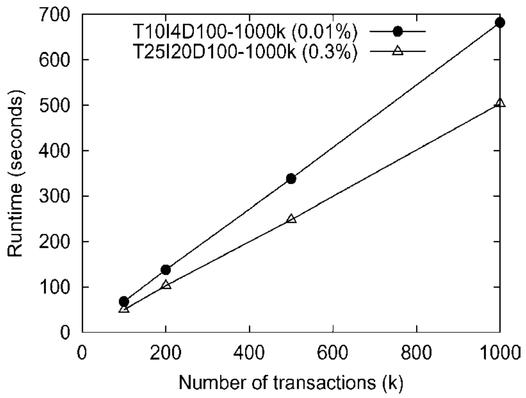


Figure 13. Scalability of *FP-growth* with number of transactions.

6.1. Materialization and maintenance of FP-trees

Although we have studied the dynamic generation of FP-trees, it is possible to materialize and incrementally update an FP-tree. We examine the related issues here.

6.1.1. Construction of disk-resident FP-trees. When the database grows very large, it is unrealistic to construct a main memory-based FP-tree. *Database projection* has been introduced in Section 3.4 as an effective approach. An interesting alternative is to construct a disk-resident FP-tree.

The B+-tree structure, popularly used in relational database systems, can be used to index FP-tree as well. Since there are many operations localized to single paths or individual item prefix sub-trees, such as pattern matching for node insertion, creation of transformed prefix paths for each node a_i, etc., it is important to cluster FP-tree nodes according to the tree/subtree structure. That is, one should (1) store each item prefix sub-tree on the same page, if possible, or at least on a sequence of continuous pages on disk; (2) store each subtree on the same page, and put the shared prefix path as the header information of the page; and (3) cluster the node-links belonging to the same paged nodes together, etc. This also facilitates a breadth-first search fashion for mining all the patterns starting from all the nodes in the header in parallel.

To reduce the I/O costs by following node-links, *mining should be performed in a group accessing mode*, that is, when accessing nodes following node-links, one should exhaust the node traversal tasks in main memory before fetching the nodes on disks.

Notice that one may also construct *node-link-free FP-trees.* In this case, when traversing a tree path, one should project the prefix subpaths of *all the nodes* into the corresponding conditional pattern bases. This is feasible if both FP-tree and one page of each of its one-level conditional pattern bases can fit in memory. Otherwise, additional I/Os will be needed to swap in and out the conditional pattern bases.

6.1.2. Materialization of an FP-tree for frequent-pattern mining. Although an FP-tree is rather compact, its construction needs two scans of a transaction database, which may represent a nontrivial overhead. It could be beneficial to materialize an FP-tree for regular frequent pattern mining.

One difficulty for FP-tree materialization is how to select a good minimum support threshold ξ in materialization since ξ is usually query-dependent. To overcome this difficulty, one may use a low ξ that may usually satisfy most of the mining queries in the FP-tree construction. For example, if we notice that 98% queries have $\xi \geq 20$, we may choose $\xi = 20$ as the FP-tree materialization threshold: that is, only 2% of queries may need to construct a new FP-tree. Since an FP-tree is organized in the way that less frequently occurring items are located at the deeper paths of the tree, it is easy to select only the upper portions of the FP-tree (or drop the low portions which do not satisfy the support threshold) when mining the queries with higher thresholds. Actually, one can directly work on the materialized FP-tree by starting at an appropriate header entry since one just need to get the prefix paths no matter how low support the original FP-tree is.

6.1.3. Incremental update of an FP-tree. Another issue related to FP-tree materialization is how to incrementally update an FP-tree, such as when adding daily new transactions into a database containing records accumulated for months.

If the materialized FP-tree takes 1 as its minimum support (i.e., it is just a compact version of the original database), the update will not cause any problem since adding new records is equivalent to scanning additional transactions in the FP-tree construction. However, a full FP-tree may be an undesirably large. Thus setting 1 as its minimum support may not be a good solution.

In the general case, we can register the occurrence frequency of every items in F_1 and track them in updates. This is not too costly but it benefits the incremental updates of an FP-tree as follows. Suppose an FP-tree was constructed based on a validity support threshold (called "watermark") $\psi = 0.1\%$ in a *DB* with 10^8 transactions. Suppose an additional 10^6 transactions are added in. The frequency of each item is updated. If the highest relative frequency among the originally infrequent items (i.e., not in the FP-tree) goes up to, say 12%, the watermark will need to go up accordingly to $\psi > 0.12\%$ to exclude such item(s). However, with more transactions added in, the watermark may even drop since an item's relative support frequency may drop with more transactions added in. Only when the FP-tree watermark is raised to some undesirable level, the reconstruction of the FP-tree for the new *DB* becomes necessary.

6.2. Extensions of frequent-pattern growth method in data mining

The philosophy of database compression and partition-based frequent-pattern mining can be extended to constraint-based mining and mining other kinds of frequent patterns, such as max-patterns, sequential patterns.

6.2.1. FP-tree mining with constraints. Constraint-based frequent-pattern mining represents an important direction towards user-controlled data mining. Constraint-based association mining using the *Apriori*-like mining methodology has been studied extensively (Srikant et al., 1997; Ng et al., 1998). With the introduction of *FP-growth* method, one may wonder whether constraint-based mining may benefit with FP-tree-like structures. A thorough study of this issue, such as classification of various kinds of constraints and development of methods of FP-tree-based mining with sophisticated constraints, such as those in Ng et al. (1998), should be the task of another research paper.[2] Here we only show how to apply FP-tree structure to mining frequent patterns by incorporation of constraints associated with a set of items.

Suppose one may just like to derive frequent patterns only associated with a particular set of items, S, such as *mining the set of frequent patterns containing c or m in Example* 1. Instead of mining frequent patterns for all the frequent items, one may explore the FP-tree-based mining as follows. With the same FP-tree, the *FP-growth* mining method may just need to be modified minorly. The only additional care is when computing a transformed prefix path for an item m, one also needs to look down the path to include the items, such as p, which are not in S. Our previous computation for the whole database will not need to consider m's pairing with p since it would have been checked when examining node p. However, since p is not in S now, such a pair would have been missed if m's computation did not look down the path to include p.

6.2.2. FP-tree mining of other frequent patterns. FP-tree-based mining method can be extended to mining many other kinds of interesting frequent patterns. We examine a few such examples.

The first example is on mining frequent closed itemsets. Since frequent pattern mining often generates a very large number of frequent itemsets, it hinders the effectiveness of mining since users have to sift through a large number of mined rules to find useful ones. An interesting alternative method proposed recently by Pasquier et al. (1999) is to mine frequent closed itemsets, where an itemset α is a closed itemset if there exists no proper superset of α that has the same support as α in the database. Mining frequent closed itemsets has the same power as mining the complete set of frequent itemsets, but it may substantially reduce redundant rules to be generated and increase the effectiveness of mining. A study at mining closed items using an *Apriori*-like philosophy but adopting a vertical data format, i.e., viewing database as "(*item_id: a set of transactions*)" instead of "(*transaction_id: a set of items*)," has been studied in Zaki and Hsiao (2002). The FP-tree-based frequent-pattern growth method can be extended and further optimized for mining such closed itemsets, which has been reported in our subsequent study, as a new closed pattern mining algorithm, called CLOSET (Pei et al., 2000).

The second example is on mining sequential patterns. A sequential patterns is a frequent pattern in an event sequence database where a sequence is a set of events happening at different times. Most of the previously developed sequential pattern mining methods, such as Agrawal and Srikant (1995), Srikant and Agrawal (1996) and Mannila et al. (1997), follow the methodology of *Apriori* since the *Apriori*-based method may substantially reduce the number of combinations to be examined. However, *Apriori* still encounters problems when a sequence database is large and/or when sequential patterns to be mined are numerous and/or long. Our frequent-pattern growth method can be extended to mining sequential patterns using the ideas of projection of sequence database and growth of subsequence fragments to confine search space. An efficient sequential pattern method, called PrefixSpan (Pei et al., 2001), has been developed in this direction and our performance study has shown a substantial performance improvement over the *Apriori*-based GSP algorithm (Srikant and Agrawal, 1996).

7. Conclusions

We have proposed a novel data structure, *frequent pattern tree* (FP-tree), for storing compressed, crucial information about frequent patterns, and developed a pattern growth method, *FP-growth*, for efficient mining of frequent patterns in large databases.

There are several advantages of *FP-growth* over other approaches: (1) It constructs a highly compact FP-tree, which is usually substantially smaller than the original database and thus saves the costly database scans in the subsequent mining processes. (2) It applies a pattern growth method which avoids costly candidate generation and test by successively concatenating frequent 1-itemset found in the (conditional) FP-trees. This ensures that it never generates any combinations of new candidate sets which are not in the database because the itemset in any transaction is always encoded in the corresponding path of the FP-trees. In this context, mining is not *Apriori*-like (*restricted*) *generation-and-test* but *frequent pattern* (*fragment*) *growth only*. The major operations of mining are count accumulation and prefix path count adjustment, which are usually much less costly than candidate generation and pattern matching operations performed in most *Apriori*-like algorithms. (3) It applies a partitioning-based divide-and-conquer method which dramatically reduces the size of the subsequent conditional pattern bases and conditional FP-tree. Several other optimization techniques, including direct pattern generation for single tree-path and employing the least frequent events as suffix, also contribute to the efficiency of the method.

We have implemented the *FP-growth* method, studied its performance in comparison with several influential frequent pattern mining algorithms in large databases. Our performance study shows that the method mines both short and long patterns efficiently in large databases, outperforming the current candidate pattern generation-based algorithms. The *FP-growth* method has also been implemented in the DBMiner system and been tested in large industrial databases, such as a retail chain database, with satisfactory performance.

Since our first publication of *FP-growth* method for mining frequent patterns without candidate generation (Han et al., 2000), there have been many subsequent studies on improvements of performance of frequent patterns based on the pattern-growth philosophy, as well as extension of the scope of the method to cover other kinds of pattern mining tasks. The pattern-growth framework has been extended towards (1) mining closed itemsets as proposed in the CLOSET algorithm (Pei et al., 2000), (2) mining sequential patterns as proposed in the PrefixSpan algorithm (Pei et al., 2001), and (3) pushing tough constraints deep into frequent pattern mining processes (Pei et al., 2001). Moreover, a notable effort is the proposal of the H-mine algorithm (Pei et al., 2001) for mining frequent patterns efficiently in sparse data sets. *FP-growth*, though efficient at mining dense data sets, may incur unnecessary overhead due to its recursive construction of FP-trees. Following the philosophy of frequent pattern growth, but not constructing FP-trees, the H-mine algorithm constructs another data structure, called *H-struct*, and mines directly on the H-struct without recursive generation of numerous conditional FP-trees. The experiments reported in Pei et al. (2001)

shows that H-mine outperforms *FP-growth* when database is sparse. A suggested approach is to integrate the two algorithms and dynamically select the FP-tree-based and H-struct-based algorithms based on the characteristics of current data distribution. Recently, some studies also show that various FP-tree mining strategies (such as bottom-up vs. top-down methods) may lead to different efficiency over data sets of different data distributions.

There are still many interesting research issues related to the extensions of pattern-growth approach, such as mining structured patterns by further development of the frequent pattern-growth approach, mining approximate or fault-tolerant patterns in noisy environments, frequent-pattern-based clustering and classification, and so on.

Acknowledgments

We would like to express our thanks to anonymous reviewers of our conference and journal submissions on this theme. Their constructive comments have improved the quality of this work.

Notes

1. Notice that *support* is defined here as *absolute* occurrence frequency, not the *relative* one as in some literature.
2. One such study has been performed by us in Pei et al. (2001).

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