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Global Discretization of Continuous Attributes as Preprocessing for Machine Learning

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ABSTRACT

Real-life data usually are presented in databases by real numbers. On the other hand, most inductive learning methods require a small number of attribute values. Thus it is necessary to convert input data sets with continuous attributes into input data sets with discrete attributes. Methods of discretization restricted to single continuous attributes will be called local, while methods that simultaneously convert all continuous attributes will be called global. In this paper, a method of transforming any local discretization method into a global one is presented. A global discretization method, based on cluster analysis, is presented and compared experimentally with three known local methods, transformed into global. Experiments include tenfold cross-validation and leaving-one-out methods for ten real-life data sets. © 1996 Elsevier Science Inc.

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1. INTRODUCTION

The process of converting data sets with continuous attributes into input data sets with discrete attributes, called *discretization*, was studied in many papers; see, e.g., [1–3, 5, 7, 10, 11, 13, 15]. We will assume that input data sets contain *examples*, characterized by *attribute* and *decision* values. A

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concept is defined as a set of all examples that have the same value w for decision d.

A data set may be either consistent or inconsistent. We need a measure of consistency for inconsistent data sets. Our measure, called a *level of consistency*, is based on *rough set theory*, a tool to deal with uncertainty, introduced by Z. Pawlak in [12]. Let U denote the set of all examples of the data set. Let P denote a nonempty subset of the set of all variables, i.e., attributes and a decision. Obviously, set P defines an equivalence relation $\wp$ on U, where two examples e and e' from U belong to the same equivalence class of $\wp$ if and only if both e and e' are characterized by the same values of each variable from P. The set of all equivalence classes of $\wp$, i.e., a partition on U, will be denoted P^*.

Equivalence classes of $\wp$ are called *elementary sets* of P. Any finite union of elementary sets of P is called a *definable set* in P. Let X be any subset of U. In general, X is not a definable set in P. However, the set X may be approximated by two definable sets in P. The first one is called a *lower approximation of X in P*, denoted by $\underline{P}X$ and defined as follows:

$$\bigcup\{Y \in P^* \mid Y \subseteq X\}.$$

The second set is called an *upper approximation of X in P*, denoted by $\bar{P}X$ and defined as follows:

$$\bigcup\{Y \in P^* \mid Y \cap X \neq \emptyset\}.$$

The lower approximation of X in P is the greatest definable set in P contained in X. The upper approximation of X in P is the least definable set in P containing X. A *rough set of X* is the family of all subsets of U having the same lower and the same upper approximations of X.

A data set is consistent with respect to decision d if and only if $\mathscr{A}^* \leq \{d\}^*$, where $\mathscr{A}$ is the set of all attributes. Furthermore, a *level of consistency*, denoted L_c, is defined as follows:

$$L_c = \frac{\sum_{X \in \{d\}^*} |\underline{\mathscr{A}}X|}{|U|}.$$

In rough set theory, the level of consistency is known as the degree of dependency of d from $\mathscr{A}$; see, e.g., [12]. For a data set that is consistent with respect to decision d, $L_c = 1$.

Let A be a continuous attribute, and let the domain of A be the interval $[a, b]$. A partition π_A on $[a, b]$ is defined as the following set of k subintervals:

$$\pi_A = \{[a_0, a_1), [a_1, a_2), \ldots, [a_{k-1}, a_k]\},$$

where $a_0 = a$, $a_{i-1} < a_i$ for $i = 1, 2, \ldots, k$, and $a_k = b$. Thus, discretization is the process that produces a partition π_A on $[a, b]$. The simplest discretization scheme is one where the discretized attribute's domain is as small as possible, i.e., $|\pi_A| = 2$. Hence the simplest (not necessarily the best) discretization is binary. We exclude the case where the size of the new domain is one (unary discretization), as the information presented in such an attribute is lost. Although there are infinitely many binary discretizations for any interval $[a, b]$, any attribute in a data set of m examples can only take on m distinct values. Hence, up to $m - 1$ binary discretizations schemes are practically possible.

The simplest method to discretize a continuous attribute is to partition its domain into equal-width intervals—the *equal-interval-width method.* A method of attribute discretization through *adaptive discretization* was proposed in [3]. The domain of an attribute is first partitioned into two equal-width intervals, and a learning system is run to induce rules. Then, the quality of the rules is tested by evaluating rule performance. If the performance measure falls below a fixed threshold, one of the partitions is subdivided further, and the process is repeated. The principal disadvantage of this method is the repetition of the learning process until the final performance level is reached.

A discretization based on maximal marginal entropy was introduced in [15]. This process involves partitioning the domain of the continuous attribute so that the sample frequency in each interval is approximately the same; it is called *equal-frequency-per-interval method.* The only parameter supplied by the user is the number of intervals to be induced on the original domain.

Another discretization, with class entropy as a criterion to evaluate a list of "best" breakpoints which together with the domain boundary points induce the desired intervals (*minimal-class-entropy method*), was suggested in [7]. A similar method of discretization is used in C4.5 [13]. The class information entropy of the partition induced by a breakpoint q is defined as

$$E(A, q; U) = \frac{|S_1|}{|U|}E(S_1) + \frac{|S_2|}{|U|}E(S_2),$$

where $E(S)$ is the entropy of the set S. The breakpoint q for which $E(A, q; U)$ is minimal among all the candidate breakpoints is taken to be the best breakpoint. This determines the binary discretization for the attribute A. In this paper, in order to induce k intervals, the procedure outlined above is applied recursively $k - 1$ times. Having computed the binary discretization on U that partitions U into two sets U_1 and U_2, we now compute binary discretizations on U_1 and U_2. Let $E(A, q_1; U_1)$ be the class information entropy associated with the best breakpoint in U_1. Let $E(A, q_2; U_2)$ be the class information entropy associated with the best breakpoint in U_2. Then, if

$$E(A, q_1; U_1) > E(A, q_2; U_2),$$

we partition U_1. Otherwise we partition U_2. Thus, the worse of the sets U_1 and U_2 is partitioned.

In this paper we propose a method to discretize attributes by using hierarchical cluster analysis. When clustering cannot be performed any further, suitable postprocessing using a class-entropy measure is performed to fuse neighboring intervals. This method of discretization is called the *cluster analysis method.*

The discretization methods presented here can be classified as either *local* or *global.* Local methods are characterized by operating on only one attribute. The methods cited above as equal interval width, equal frequency per interval, and minimal class entropy are local, while cluster analysis is global. Global methods are characterized by considering *all* attributes (rather than one) before making a decision where to induce interval breakpoints. Section 2 on the globalization of local discretization methods discusses the steps we have taken to improve local methods by minimizing user interaction.

2. GLOBALIZATION OF LOCAL DISCRETIZATION METHODS

Local methods suffer from the inability to predict how many intervals should be induced for a given domain of a continuous attribute A. Often, only an expert can tell accurately into how many intervals the domain of A should be partitioned and how the partitioning ought to be done. Being unaware of how many intervals to induce, which method to use, or how many times to perform the discretization for each continuous attribute can seriously jeopardize the outcome of the discretization process. We can, however, suggest the following guidelines that seem likely to insure successful discretization:

- *Complete discretization.* We are seldom interested in discretization of just one continuous attribute (unless there is only one such attribute in a data set).
- *Simplest result of discretization.* In general, the smaller the size of an attribute's domain after discretization, the simpler the rules that are induced from discretized data. As a result, the knowledge encompassed by such an attribute is more general.

- *Consistency.* Data sets with continuous attributes are almost always consistent ($L_c = 1$). When a discretization scheme applied to an attribute's values is "bad," an inconsistent data set may be obtained. When this happens, we lose valuable information. We should keep the level of consistency of the new discretized data set as close as possible to that of the original data.

With these points in mind, the first step in transforming local discretization method into a global one is as follows. Using a chosen method (e.g., equal-interval-width discretization, equal-frequency-per-interval discretization, minimal-class-entropy discretization), we partition each continuous attribute's domain into two intervals (binary discretization). Thus we achieve two out of three goals: complete and simplest discretization. At this time, exactly one of the following is true: (a) $L_c^D = L_c$ or (b) $L_c^D < L_c$, where L_c^D is the level of consistency of the new discretized data set and L_c is the level of consistency of the original data set.

If (a) is true, we have produced the simplest discretization possible, discretized all candidate attributes, and maintained the original consistency level, thereby minimizing information loss due to discretization. If (b) is true, we need to do more work. It is necessary to rediscretize the attribute's domain whose initial (binary) discretization was "poorest." Rediscretization will involve inducing $k + 1$ intervals on the domain of an attribute if k are currently present (using the same local discretization method). Determining which attribute's domain exhibits the "poorest" partitioning, however, is not straightforward. One way to do it is to play a series of what-if scenarios for each attribute. In turn, we compute the gain in the level of consistency of the data set as each attribute is rediscretized from k to $k + 1$ intervals. The attribute whose rediscretization produces the highest gain in level of consistency of the information system is selected and rediscretized. While this is an intuitively attractive method, it requires many time-consuming computations of the level of consistency to play out the various scenarios.

We propose a method to measure the performance of an attribute (after discretization) based on the class entropy of attribute values. The rationale for using class entropy comes from the fact that if for some block $B \subseteq \{A\}^*$ the class entropy of B is zero, then there exists a concept C in $\{d\}^*$ such that $B \subseteq C$, i.e., the block B is sufficient to describe (in part or whole) the concept C. This indicates that the current partitioning is good at least with respect to the block B.

The higher the entropy of the block B, the more randomness is encountered with respect to a concept and hence the greater the chance of "poor" discretization. Since we are looking at whole attributes rather than just blocks of attribute-value pairs, we will compute the average block entropy of an attribute A. We compute this measure according to the following formula:

$$\mathscr{M}_{\{A^D\}^*} = \frac{\sum_{B \in \{A^D\}^*} (|B|/|U|) E(B)}{|\{A^D\}^*|},$$

where $\{A^D\}^*$ is the partition induced by the discretized attribute A^D. A candidate attribute for which $\{A^D\}^*$ is maximum is selected as the next attribute for rediscretization. The merit of computing the average block entropy for an attribute is that we need only recompute this measure for the attribute which was last picked for rediscretization. It is not a computationally intensive procedure.

3. DISCRETIZATION BASED ON CLUSTER ANALYSIS

The purpose of cluster analysis is to search for similar objects and group them into classes, or clusters [6]. Objects are points in n-dimensional space which are defined by n characteristic values. During agglomerative cluster formation, objects that exhibit the most similarity are fused into a cluster. Once this process is completed, clusters can be analyzed in terms of all attributes.

The intent of the first step of this discretization method (cluster formation) is to determine initial intervals on the domains of the continuous attributes. During the second step (postprocessing) we hope to minimize the number of intervals that partition the domain of each continuous attribute.

3.1. Cluster Formation

There exist many different clustering techniques, none of which is a solution to all potential clustering needs. For our purposes, we have elected to use the *median cluster analysis* method.

Let $m = |U|$, and let $\{A_1, A_2, \dots, A_i, A_{i+1}, \dots, A_n\}$ be the set of all attributes, where attributes $A_1, A_2, \dots, A_i$ are continuous and attributes $A_{i+1}, A_{i+2}, \dots, A_n$ are discrete $(1 < i \leq n)$. Let $e \in U$. We define the continuous component of e as

$$e_{\text{continuous}} = (x_1^e, x_2^e, \dots, x_i^e)$$

and the discrete component of e as

$$e_{\text{discrete}} = (x_{i+1}^e, x_{i+2}^e, \dots, x_n^e).$$

In cases where continuous attributes' values are not of the same scale (foot, pounds, light-years, etc.), we must standardize attribute values to zero mean and unit variance for clustering to be successful. This is done by dividing the attribute values by the corresponding attribute's standard deviation (derived from the complete set of values for the attribute) [6].

We begin cluster formation by computing an $m \times m$ distance matrix between every pair of continuous components of examples in U. The entries in the distance matrix correspond to squared Euclidean distances between data points in i-dimensional space. We initiate clusters by allowing each i-dimensional data point to be a cluster. Thus, we originally have m clusters, each of cardinality one.

New clusters are formed by merging two existing clusters that exhibit the most similarity. In our case, this involves finding two clusters that are separated by the smallest Euclidean distance. When such a pair is found (clusters b and c), they are fused to form a new cluster bc. The formation of the cluster bc introduces a new cluster into the space, and hence its similarity (distance) to all the other remaining clusters must be recomputed. For this purpose, we use the *Lance-Williams flexible method* [6]. Given a cluster a and a new cluster bc to be formed from clusters b and c, the distance from bc to a is computed as

$$d_{a(bc)} = d_{(bc)a} = \alpha_b d_{ab} + \alpha_c d_{ac} + \beta d_{bc} + \gamma |d_{ab} - d_{ac}|,$$

where $\alpha_b = \alpha_c = \frac{1}{2}$, $\beta = \frac{1}{4}$, and $\gamma = 0$ for the median cluster analysis method.

At any point during the clustering process the clusters formed induce a partition on the set of examples U. Examples that belong to the same cluster are indiscernible by the subset of continuous attributes. Therefore, we should continue cluster formation until the level of consistency of the partition $\{K \mid K \text{ is a cluster}\}$ is equal to or greater than the original data's level of consistency L_c.

When the above condition fails, cluster formation stops, and we analyze the clusters to determine candidate intervals that will partition the domain of each of the i continuous attributes. Since the points in the clusters are defined by i attribute values, we can examine how the individual attributes define each cluster. Let r be the number of clusters produced. Let K be a cluster. The set of data points of an attribute A_j $(1 \leq j \leq i)$ that define a cluster K is

$$\mathrm{DP}_{A_j}^{K} = \{x_j^e \mid e \in K\}.$$

Hence, the defining interval I_{K,A_j} of cluster K_j with respect to attribute A_j is

$$I_{K,A_j} = \left[L_{K,A_j}, R_{K,A_j}\right] = \left[\min\left(\mathrm{DP}_{A_j}^{K}\right), \max\left(\mathrm{DP}_{A_j}^{K}\right)\right].$$

It is possible that for a given attribute A_j, the domain that defines the cluster K is a subdomain of a domain that defines another cluster K', i.e., $L_{K,A_j} \geq L_{K',A_j}$ and $R_{K,A_j} \leq R_{K',A_j}$. From the perspective of the intervals

$$I_{K,A_j} = \left[L_{K,A_j}, R_{K,A_j}\right] \quad \text{and} \quad I_{K',A_j} = \left[L_{K',A_j}, R_{K',A_j}\right]$$

we can safely eliminate the subinterval I_{K,A_j} from further consideration without compromising the discretization outcome.

At this time we are ready to induce intervals on each attribute's continuous domain. For each attribute A_j where $j \in \{1,2,\ldots,i\}$ we construct two sets, L_{A_j} and R_{A_j}, which contain the left and right boundary points respectively of the defining intervals I_{K_l,A_j} for $l \in \{1,2,\ldots,r\}$. Hence,

$$L_{A_j} = \left\{L_{K_l,A_j} \middle| l \in \{1,2,\ldots,r\}\right\}$$

and

$$R_{A_j} = \left\{R_{K_l,A_j} \middle| l \in \{1,2,\ldots,r\}\right\}.$$

The interval partition on the domain of the attribute A_j is equal to

$$\pi_{A_j} = \left\{\left[\min_1(L_{A_j}), \min_2(L_{A_j})\right), \left[\min_2(L_{A_j}), \min_3(L_{A_j})\right), \ldots, \left[\min_r(L_{A_j}), \max(R_{A_j})\right]\right\},$$

where $\min_a(L_{A_j})$ corresponds to the ath smallest element of L_{A_j}.

3.2. Postprocessing

Postprocessing involves merging adjacent intervals in an attempt to reduce the domain size of each of the discretized attributes. This final processing task is divided into two stages: "safe" merging of intervals and merging of intervals.

Given an interval partition on the domain of an attribute A_j we can "safely" merge adjacent intervals provided that the outcome does not affect in any way the accuracy of the information system. Let

$$\pi_{A_j} = \{[a_0, a_1), [a_1, a_2), \ldots, [a_{k-1}, a_k]\}$$

be the interval partition of attribute A_j. Consider any pair of adjacent intervals $[a_{l-1}, a_l)$ and $[a_l, a_{l+1})$. We can fuse them safely into one interval $I_{l-1,l+1} = [a_{l-1}, a_{l+1})$ if and only if the class entropy of the block associated with $I_{l-1,l+1}$ is 0. A class entropy value of 0 indicates that the new interval $I_{l-1,l+1}$ describes one concept only (in part or in full). Therefore, the consistency of the data set is not violated. Safe merging of intervals is applied sequentially to all attributes for which intervals have been induced. Merging involves exactly the same task as the previous step. However, since each of the merges performed may affect the consistency of the data set, two questions must be resolved first:

1. which attribute's intervals to merge first, and
2. which neighboring intervals to merge first.

In order to solve these problems we utilize the class entropy function to determine the priority of interval merging. The rationale behind this choice is that we are interested in forming intervals whose blocks exhibit most class uniformity. Intervals whose blocks show high class entropy are not desirable, because they may contribute adversely to the rule induction process.

To prioritize interval merging, we compute the class entropy of the block associated with every pair of adjacent intervals for all continuous attributes. We pick a pair of adjacent intervals for merging whose class entropy is the smallest amongst continuous attributes. By employing this procedure, we are not biasing merging to any particular attribute, and hence point 1 above is resolved. There may be some natural bias, however, especially if data contained in an attribute show high class correlation.

Before a merge is performed, we must check if the accuracy of the data set will fall below a given threshold as a result of the merge. If the accuracy will still be adequate, we perform the merge. Otherwise, we mark this pair of adjacent intervals as nonmergeable and proceed to the next candidate. The process stops when each possible pair of adjacent intervals is marked as nonmergeable.

4. EXPERIMENTS

In order to compare the performance of discretization methods presented in this report, we have subjected a sample of ten data sets to discretization. The data contain real-life information from fields such as medicine, insurance, banking, and science and have been used previously in testing pattern recognition and machine learning methods. Table 1 gives a summary of data sets used in experiments.

The data sets *GM*, *rocks*, and *bank* were donated by W. Koczkodaj from Laurentian University, Canada. The first data set contains financial data gathered by General Motors; the second, information about volcanic rocks. The data set *bank*, describing bankruptcy data, was created by E. Altman and M. Heine at the New York University School of Business in 1968. R. A. Fisher is credited for the well-known *iris* data set. The data set *hsv-r*,

Table 1. Data Sets

Data set	Number of examples	Number of continuous attributes	Number discrete attributes	Number of concepts
GM	200	20	0	2
rocks	1133	13	0	4
iris	150	4	0	3
bank	66	5	0	2
hsv-r	122	9	2	4
bupa	345	6	0	2
glass	214	9	0	7
wave	512	21	0	3
image	210	19	0	7
cars	193	14	9	6

donated by R. Slowinski from Technical University of Poznan, Poland, represents raw data on treatment of duodenal ulcer by HSV. The remaining five data sets, as well as *iris*, were taken from the University of California at Irvine repository of machine learning databases. The data set *bupa*, describing a liver disorder, contains data gathered by BUPA Medical Research Ltd, England. The data set *glass*, representing glass types, has been created by B. German, Central Research Establishment, Home Office Forensic Science Service, Canada. The data set *waveform*, as described in [1], represents three types of waves. The data set *image*, created in 1990 by the Vision Group, University of Massachusetts, represents image features: brickface, sky, foliage, cement, window, path, and grass. The data set *cars*, created in 1985 by J. C. Schlimmer, represents insurance risk ratings. All of the above ten data sets have level of consistency equal to 100%.

Each of the data sets in the sample was discretized completely, i.e., all continuous attributes were processed. We have used the globalized version of the local methods equal interval width and equal frequency per interval, as well as the cluster analysis method, to perform discretization. Furthermore, in order to minimize information loss due to discretization, we have maintained the original consistency level throughout the various discretization procedures. Hence, we have produced four consistent data sets from each data set in the sample.

As discretization is only a preprocessing step to concept acquisition through machine learning, it seemed worthwhile to investigate the quality of the knowledge base induced from the preprocessed data sets. To make this possible we have utilized the learning program LEM2 (Learning from

Examples Module, version 2) to induce rules from the discretized data sets. LEM2 is a typical member of the family of learning algorithms in that it finds a minimal discriminant description; see, e.g., [4, 8].

The most important performance criterion of the discretization method is the accuracy rate. A complete discussion on how to evaluate the error rate (and hence the accuracy rate) of a rule set induced from a data set is contained in [14]. We have used the following cross-validation guidelines in computing the accuracy rate:

- If the number of examples was less than 100, the leaving-one-out method was used to estimate the accuracy rate of the rule set. This method involves m learn-and-test experiments (where $m = |U|$). During the jth experiment, the jth example is removed from the data set, automatic concept acquisition is performed on the remaining $m - 1$ examples (using LEM2), and the classification of the omitted example by rules produced is recorded. The accuracy rate is computed as

$$1 - \frac{\text{total number of misclassifications}}{\text{total number of examples}}.$$

- If the number of instances in the data set was more than 100, the tenfold technique was used. This technique is similar to leaving one out in that it follows the learn-and-test paradigm. In this case, however, the learning sample corresponds to 90% of the original data, the testing sample is the remaining 10%, and the experiments are repeated ten times. This method is used primarily to save time at negligible expense in accuracy. The accuracy rate is computed in the same way as for the leaving-one-out method.

5. CONCLUSIONS

To analyze the results obtained (see Table 2), we have used the Wilcoxon matched-pairs signed-rank test; see, e.g., [9, pp. 546–561]. The purpose of this nonparametric test is to determine if significant differences exist between two populations. Paired observations from the two populations are the basis of the test, and magnitudes of differences are taken into account. This is a straightforward procedure to either accept or reject the null hypothesis, which is commonly taken to be identical population distributions.

The cluster analysis method outperformed the equal-interval-width and equal-frequency-per-interval methods. The cluster analysis method is better than equal-interval-width method with 1% significance level for a one-tailed test. Also, the cluster analysis method is better than the equal-

Table 2. Accuracy Rate after Discretization

Data set	Equal interval width	Equal frequency per interval	Minimal class entropy	Cluster analysis
GM	68.0	59.0	73.0	69.0
rocks	57.5	54.2	55.6	53.0
iris	91.5	86.7	82.0	95.3
bank	77.3	95.5	84.9	97.0
hsv-r	42.5	35.8	46.7	48.3
bupa	41.9	39.7	41.3	42.5
glass	54.7	49.5	56.1	60.3
wave	99.4	99.4	99.4	99.8
image	69.0	70.0	73.8	77.6
cars	58.0	59.6	67.8	63.7

frequency-per-interval method with 0.5% significance level for a one-tailed test. Minimal class entropy showed no significant performance difference from any of the other methods, i.e., the null hypothesis that minimal class entropy performs like any other method could not be rejected even at the 5% significance level for a one-tailed test.

In view of these results, we would like to suggest that more study should be done to better understand how clustering-based techniques can be used in discretization. Admittedly, we have selected one of the most straightforward clustering methods. It would be worthwhile to examine how different clustering methods perform in this global approach to discretization.

In the suggested globalization of local discretization methods, the search for a new attribute to be rediscretized is based on the maximum of the measure $M_{\{A^D\}^*}$. This way the search is oriented toward looking for the worst attribute that should be modified. Other criteria should be investigated as well, e.g., looking for the best attribute.

Finally, it is likely that certain local discretization methods work better with some types of data. It would be interesting to see whether a determination could be made to select a given discretization method based solely on the data characteristics of an attribute or a data set.

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